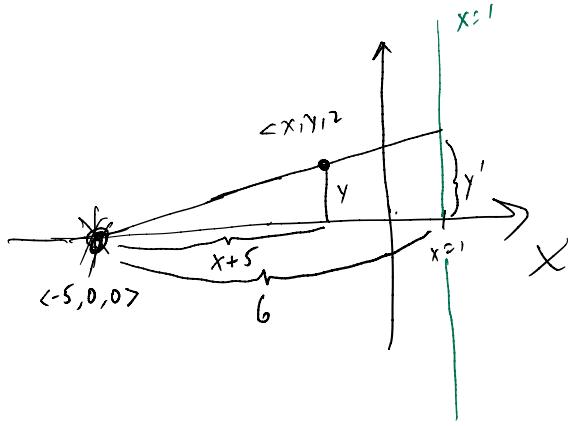
Math 155A - Spring 2022 - Quiz #7 - April 21

Name: Answer Key

PID:

1. A light source is placed at $\langle -5, 0, 0\rangle$ and it casts shadows onto the yz-plane P defined by $x = 1$. The $x = 1$ plane is parallel to the yz-plane and acts like an infinite wall.
When $\langle x, y, z\rangle$ is a point in $\mathbb{R}^3$ with $-5 < x \leq 1$, define $A(\langle x, y, z\rangle)$ to be the position of the shadow of the point on the plane P. E.g, $A(\langle -2, 1, 2\rangle) = \langle 1, 2, 4\rangle$, and $A(\langle -3, -1, -2\rangle) = \langle 1, -3, -6\rangle$.

(a) Working in ordinary coordinates (not homogeneous) give the formula expressing the mapping $A(\langle x, y, z\rangle) = \langle x', y', z'\rangle$. That is, give formulas for x', y', z' in terms of x, y, z.



$x' = 1$

$\frac{y'}{6} = \frac{y}{x+5}$ So $y' = \frac{6y}{x+5}$

Similarly $z' = \frac{6z}{x+5}$

So $A: \langle x,y,z\rangle \longmapsto \langle 1, \frac{6y}{x+5}, \frac{6z}{x+5}\rangle$

(b) Give a 4×4-matrix that represents the transformation A over homogeneous coordinates.

In homogeneous coordinates,

$\langle x, y, z, 1\rangle \longmapsto \langle x+5, 6y, 6z, x+5\rangle$

4x4 Matrix : $\begin{pmatrix} 1 & 0 & 0 & 5 \\ 0 & 6 & 0 & 0 \\ 0 & 0 & 6 & 0 \\ 1 & 0 & 0 & 5 \end{pmatrix}$.