**Announcements**

"Coffee hour" tomorrow @ 10:30
Project 4 due tomorrow (or Sunday)

<table>
<thead>
<tr>
<th>Grade Distribution</th>
<th>#</th>
</tr>
</thead>
<tbody>
<tr>
<td>100 - 120</td>
<td>23</td>
</tr>
<tr>
<td>80 - 100</td>
<td>18</td>
</tr>
<tr>
<td>60 - 80</td>
<td>17</td>
</tr>
<tr>
<td>40 - 60</td>
<td>17</td>
</tr>
<tr>
<td>&lt; 40</td>
<td>2</td>
</tr>
</tbody>
</table>
Two triangles

Each pixel should belong to exactly one of the triangles.

A pixel belonging to both will mess up transparency.

Render an array of pixels 

Pixels on boundary between \( \triangle yz \) and \( \triangle xz \) (blue)
Render a surface of revolution—by using a triangle strip that is rotated and rendered multiple times, will cause problems from roundoff errors.

Same strip rotated.

Round off errors can make common vertices not quite coincide.

If compute vertices with $\Theta = 0$ and $\Theta = 2\pi$ to finish, with limited precision:

$$\sin(0) \neq \sin\left(\frac{1}{2\pi}\right)$$
Related issue

Risk of pixels not correctly being in exactly one triangle.

Fix: Split $\Delta xyz$ with 2 triangles namely $\Delta xvy$ and $\Delta xzy$
Overview of computing surface normals: Three methods

Goal: Render a set of triangles to generate a smooth looking surface

Add normals to help us render lighting

Method #1: Approximation to a surface

Use normals for the triangles

Average normal of adjacent triangles to estimate the normal on a vertex

N = \sum_n \vec{n}_i

Assume \vec{n}_i's are unit vectors
Possibly normalize \vec{n}, i.e., make it to unit vector.
Research topic?

Question Boundary
For mathematically defined surface $H: \mathbb{R}^3 \to \mathbb{R}$, we have:

1. Gradient method
   \[ \nabla H \text{-gradient} \]

2. Cross product of partial derivatives method
   \[ \hat{p} = \hat{p}(u,v) \]
   \[ \frac{\partial \hat{p}}{\partial u} \times \frac{\partial \hat{p}}{\partial v} \]

Surface of revolution
\[ \hat{p}(v, \theta) = \langle v \sin \theta, h(v), v \cos \theta \rangle \]
Transforming Normals:

Example in $\mathbb{R}^2$

$\langle 1, 2 \rangle$ is tangent to the surface $\langle 1, 2 \rangle \cdot \langle 2, 1 \rangle = 0$

Slope $-2$,

Normal vector: $\langle 2, 1 \rangle$ (for example)

Did not form $S_{\langle 1, \frac{1}{2} \rangle} \langle 2, 1 \rangle = \langle 2, \frac{1}{2} \rangle$

Transform by $S_{\langle 1, \frac{1}{2} \rangle}$ (scaling)

Slope $-1$

New normal vector: $\langle 1, 1 \rangle$

Let $M$ be the $3 \times 3$ matrix for the transformation.

Normal transforms by $n \mapsto (MT)^{-1}n$
Phong lighting

**Types of lighting**

- **Ambient light** (directionless)
- **Diffuse light** (incoming direction, reflects in all directions)
- **Specular** - "almost" mirror-like reflection

- Light source
- Incoming light
- Outgoing light reflected off the surface
- Viewer/camera

Take into account:

- Direction of the incoming light \( \mathbf{l} \)
- Surface normal \( \mathbf{n} \)
- Direction to viewer \( \mathbf{v} \)
- Color/brightness of the light
- Reflectivity of the surface.