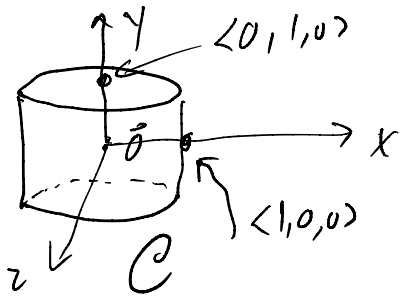
Earth/Solar system used perspective

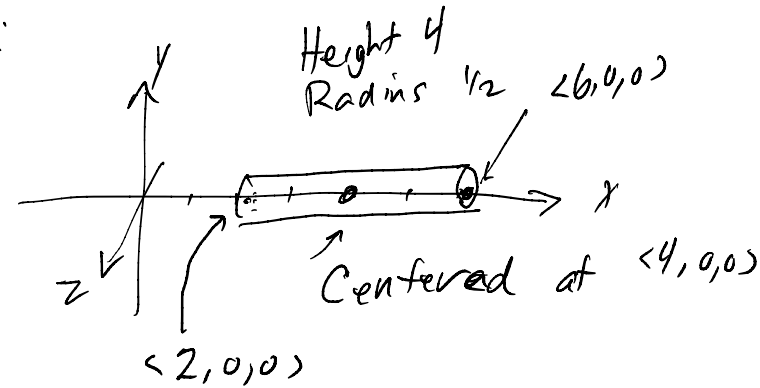
Tents did not use perspective – instead used orthographic projections.

Recall Example from last lecture:

Height 2
Radius 1
cylinder
Centered at $\vec{0}$



$\xrightarrow{f}$

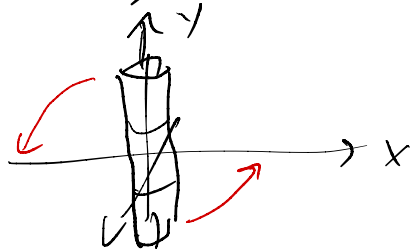


Express f as a composition of scalings, translations and rotations

$S_{\langle 1/2, 2, 1/2 \rangle}$

Lets form f by: first scale, then rotate, then translate.

$S_{\langle 1/2, 2, 1/2 \rangle}(C)$

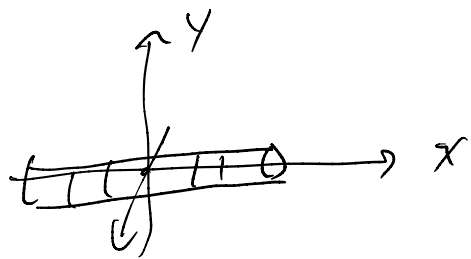


Height 4
Radius $1/2$

C = cylinder

$R_{\theta, \vec{u}}$ - rotates around axis $\vec{u}$ through $\vec{0}$.

Lets $R_{90^\circ, \vec{k}}$



$T_{\langle 4,0,0 \rangle}$ - does the required translation.

Express f as $T_{\langle 4,0,0 \rangle} \circ R_{90^\circ, \vec{k}} \circ S_{\langle 1/2, 2, 1/2 \rangle}$

In pseudocode

$M := \text{Identity}(\text{ViewMatrix})$

$M := M \cdot T_{\langle 4,0,0 \rangle}$

$M := M \cdot R_{90^\circ, \vec{k}}$

$M := M \cdot S_{\langle 1/2, 2, 1/2 \rangle}$

$\cdot$ means matrix multiplication

M is a 4×4 matrix.

In C# Code

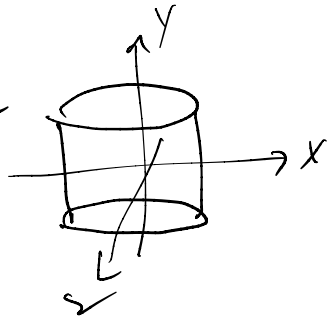
```
M.MultGlTranslate(4,0,0);
M.MultGlRotate(π/2, 0,0,1);
M.MultGlScale(1/2, 2, 1/2);
```

<u>Advantage of Multiplying on the right:</u>

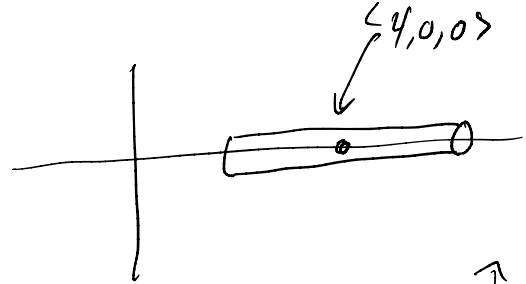
Supports hierarchical application of transformations.

Let's f as by: First Rotate, Then Scale, then Translate

Height 2
Radius

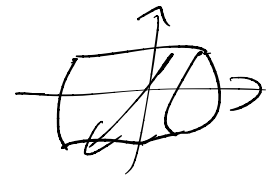


$\Rightarrow$

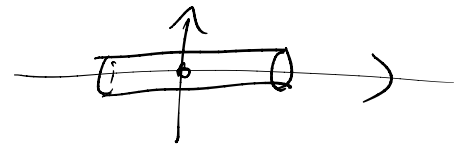


Height 4
Radius 1/2

Rotate $R_{90°, \vec{k}}$



Scale $S_{\langle 2, 1/2, 1/2 \rangle}$

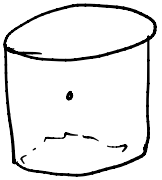


then Translate $T_{\langle 4,0,0 \rangle}$

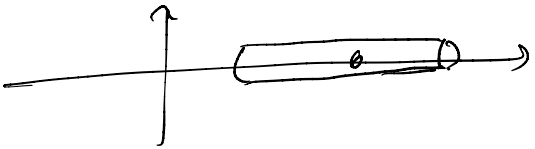
$S_{\langle 1/2, 2, 1/2 \rangle}$ doesn't work anymore

Now express f by scaling first then translating then rotating

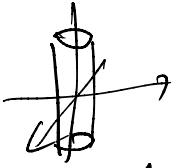
Height 2
Radius 1



$\Longrightarrow$

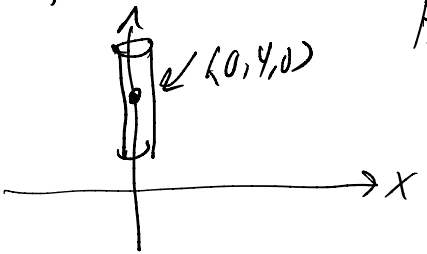


first $S_{\langle 1/2, 2, 1/2 \rangle}$



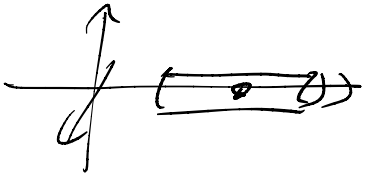
Height 4

Translate $\langle 0,4,0 \rangle$



$$\boxed{f = R_{90^\circ,k} \circ T_{\langle 0,4,0 \rangle} \circ S_{\langle 1/2, 2, 1 \rangle}}$$

Then rotate -90 around z axis $R_{-90,\vec{k}}$



$$R_{-90,\vec{k}} = R_{90^\circ,-\vec{k}}$$

$R_{\theta, \vec{u}}$ where $\vec{u}$ not $\pm\vec{i}$ or $\pm\vec{j}$ or $\pm\vec{k}$

<u>Example</u>:

$R_{180^\circ, \vec{i}+\vec{k}}$

$$\vec{i} \longmapsto \vec{k}$$
$$\vec{j} \longmapsto -\vec{j}$$
$$\vec{k} \longmapsto \vec{i}$$

<u>Theorem</u>: (Euler's): Any rigid, orientation preserving linear map is equal to $R_{\theta, \vec{u}}$ for some θ, $\vec{u}$.

<u>Example</u>: Let f be linear, permuting the axes, so that $f(\vec{i}) = \vec{j}$ $f(\vec{j}) = \vec{k}$ and $f(\vec{k}) = \vec{i}$.

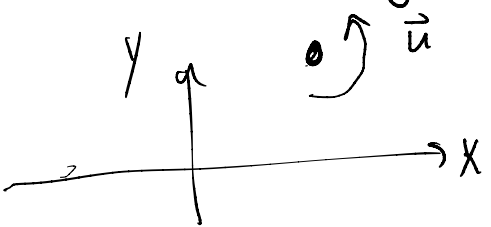
Express f as a rotation $R_{\theta,\vec{u}}$.

Take $\vec{u} = \vec{i} + \vec{j} + \vec{k}$ (or $\frac{1}{\sqrt{3}}(\vec{i}+\vec{j}+\vec{k})$ is a unit vector)

Take $\theta = \frac{2\pi}{3}$ or $120°$.

Related theorem in $\mathbb{R}^2$

<u>Thm</u> Any rigid, orientation preserving affine map is either a generalized rotation or a translation



$R_\theta^{\vec{u}}$

Holds $\vec{u}$ fixed
Rotates around $\vec{u}$ angle θ (in ccw direction).