Earth/Solar system used perspective. Tents did not use perspective — instead used orthographic projections.

Recall Example from last lecture:

Height 2
Radius 1

cylinder centered at \( \hat{0} \)

Express \( f \) as a composition of scalings, translations, and rotations:

\[ S < \frac{1}{2}, 2, \frac{1}{2} \]
Let's form $f$ by: first scale, then rotate, then translate.

$$S_{\left( \frac{1}{2}, \frac{1}{2}, \frac{1}{2} \right)}(C)$$

$C = \text{cylinder}$

$R_{\theta, \hat{u}} \cdot \text{rotates around axis } \hat{u} \text{ through } \hat{0}.

$$T_{<4,0,0>} \cdot \text{over the required translation.}$$

Express $f$ as

$$T_{<4,0,0>} \circ R_{\theta, \hat{u}} \circ S_{\left( \frac{1}{2}, \frac{1}{2}, \frac{1}{2} \right)}$$
In pseudocode:

\[
M := \text{Identity (ViewMatrix)}
\]

\[
M := M \cdot T(4,0,0)
\]

\[
M := M \cdot R_{90, x}
\]

\[
M := M \cdot S(\frac{1}{2}, 2, \frac{1}{2})
\]

* means matrix multiplication.

\[M\] is a 4x4 matrix.

Advantage of Multiplying on the right:

Supports hierarchical application of transformations.

In C++ Code:

\[
\text{M. Mult-gl Translate (4,0,0);}
\]

\[
\text{M. Mult-gl Rotate (\pi/2, 0,0,1);}
\]

\[
\text{M. Mult-gl Scale (1/2, 2, 1/2);}
\]
Let's $f$ as by: First Rotate, then Scale, then Translate.

Rotate $R_{90^\circ}$

Scale $S_{(2,\frac{1}{2},\frac{1}{2})}$

then Translate $T_{(4,0,0)}$.
Now express $f$ by scaling first, then translating, then rotating.

**First:**

$S(\frac{1}{2}, 2, \frac{1}{2})$

**Translate:**

$<0, 4, 0>$

Then rotate $-90^\circ$ around $z$ axis:

$R_{-90, \hat{k}} = R_{90^\circ, -\hat{k}}$

**Height:**

$2$

**Radius:**

$1$
Example:

\[ R_{180^\circ}, \vec{z} + \vec{k} \]

\[ \vec{i} \rightarrow \vec{k} \]
\[ \vec{j} \rightarrow -\vec{j} \]
\[ \vec{k} \rightarrow \vec{i} \]

Theorem (Euler's): Any rigid, orientation-preserving map is equal to \( R_\theta, \vec{u} \) for some \( \theta, \vec{u} \).

Example: Let \( f \) be linear, permuting the axes, so that \( f(\vec{i}) = \vec{j} \) \( f(\vec{j}) = \vec{k} \) and \( f(\vec{k}) = \vec{i} \).
Express \( f \) as a rotation \( R\theta, \bar{u} \).

Take
\[
\bar{u} = i + j + k \quad \text{(or} \quad \frac{1}{\sqrt{3}} (i + j + k) \text{)}
\]

is a unit vector.

Take \( \theta = \frac{2\pi}{3} \) or \( 120^\circ \).

---

Related theorem in \( \mathbb{R}^2 \)

Thus, any rigid, orientation-preserving affine map is either a generalized rotation or a translation.

\[ y \begin{array}{c} \rightarrow \end{array} \Phi \bar{u} \]

\[ \begin{array}{c} x \end{array} \]

Holds \( \bar{u} \) fixed. Rotates around \( \bar{u} \) angle \( \theta \) (in CCW direction).