See blackboard for earlier material

\[ M = \begin{pmatrix} 0 & 2 & 0 \\ -\frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} \end{pmatrix} \]

Let \( T_{\mathbf{u}} \mathbf{x} = \mathbf{x} + \mathbf{u} \)

So \( T_{\mathbf{u}} \) is a translation.

Define \( h \) as a composition involving \( f \) and \( T_{\mathbf{u}} \).

Is \( h \) equal to \( T_{\mathbf{u}} f \) or to \( f \circ T_{\mathbf{u}} \)?
Try Tu of. (Yes!)

What does $T_u$ do to the cylinder?

$T_u \begin{array}{c}
\begin{array}{c}
\text{cylinder}
\end{array}
\end{array} \text{ to } \begin{array}{c}
\begin{array}{c}
\text{cylinder}
\end{array}
\end{array}$

$f(4,0,0) = (0, -2, 0)$
Uniform scaling $S_\alpha(\vec{x}) = \alpha \vec{x}$ $\alpha \in \mathbb{R}$

Non-uniform scaling $S_{\alpha, \beta, \gamma}(\langle x_1, x_2, x_3 \rangle) = \langle \alpha x_1, \beta x_2, \gamma x_3 \rangle$

Rotations $R_{\theta, \vec{u}}$ - rotates angle $\theta$ around axis $\vec{u}$, direction given by right-hand rule.

Example $R_{90, \hat{j}}$ - is represented by the $3 \times 3$ matrix:

$$
\begin{pmatrix}
0 & 0 & 1 \\
0 & 1 & 0 \\
-1 & 0 & 0
\end{pmatrix}
$$
Try $S_{(1,1,1)}$ and $R_{90, \bar{j}}$,

What are $S_{(1,1,1)} \circ R_{90, \bar{j}}$

and $R_{90, \bar{j}} \circ S_{(1,1,1)}$?

How do act on a unit sphere centered at the origin?

Unit sphere

This is $S_{(1,1,1)} \circ R_{90, \bar{j}}$

Smushed in the x-direction
Think of $(S_{\frac{1}{2}, 1, 1} \circ R_{\theta, \frac{\pi}{2}})^2 x = S_{\frac{1}{2}, 1, 1} (R_{\theta, \frac{\pi}{2}} (x))$

What would $R_{\theta, \frac{\pi}{2}} \circ S_{\frac{1}{2}, 1, 1}$ do?

Now smashed in the $z$ direction.

$S_{\frac{1}{2}, 1, 1}$

$\rightarrow$

$R_{\theta, \frac{\pi}{2}}$

$\rightarrow$

Smush in the $x$ direction.
In fact $R_{90^\circ_j} \circ T(1,0,0)$ is equal to $T(0,0,-1)^\circ R_{90^\circ_j}$

$T < 0, 0, 1 >$ - moving (translating) one unit in z-direction

Front of sphere is touching the origin.
Center of the sphere is at $<0, -1, 0>$.
How to represent this by a matrix?

Use homogeneous coordinates.

Use a $4 \times 4$ matrix

$$\vec{x} \rightarrow M \vec{x} + \vec{t}$$

$$
\begin{pmatrix}
M & \vec{t} \\
0 & 1
\end{pmatrix}
$$

$T_{(0,0,15^\circ)}^\mathbb{R}$ has $4 \times 4$ matrix

$$
\begin{pmatrix}
0 & 0 & 1 & 0 \\
0 & 1 & 0 & 0 \\
-1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1
\end{pmatrix}
$$

Translation $\vec{t} = \vec{1}$
In open GL:

Set \( M = \text{Identity} \)

\[
M = M \circ T_{\langle 0,0,-1 \rangle}
\]

\[
M = M \circ R_{90,j}
\]

This makes \( M = T_{\langle 90,-0 \rangle} \circ R_{90,j} \)

+ it can be used to render the sphere transformed by \( M \),

\[
M \hat{x} = R_{90,j} \hat{x} + \langle 0,0,-1 \rangle = T_{\langle 0,0,-1 \rangle}(R_{90,j}(x)).
\]