

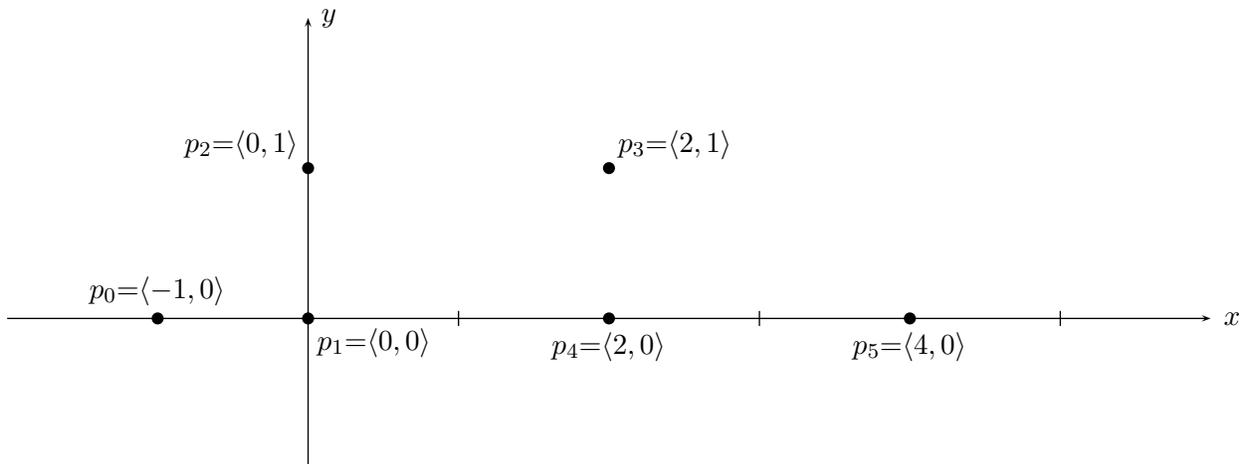
11. (Degree 3 Bézier curve.) A particle is animated in  $\mathbb{R}^3$  so that it starts at  $\langle 1, 0, 1 \rangle$  at time  $u = 0$  and arrives at  $\langle 5, 2, 4 \rangle$  at time  $u = 1$ . The initial velocity (at time  $u = 0$ ) is equal to  $\langle 6, 0, 0 \rangle$ , and its final velocity (at time  $u = 1$ ) is equal to  $\langle 0, 0, 3 \rangle$ . The motion is modeled with a degree three Bézier curve  $\mathbf{q}(u)$ .

- Give the control points  $\mathbf{p}_0, \mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3$  for the Bézier curve  $\mathbf{q}(u)$ .
- Use the de Casteljau method to find the position of the particle at time  $u = \frac{1}{2}$ . (Show your work, including the values of  $\mathbf{r}_0, \mathbf{r}_1, \mathbf{r}_2, \mathbf{s}_0, \mathbf{s}_1, \mathbf{t}_0$ .)
- The “first half” of the Bézier curve is the curve  $\mathbf{q}_1(u) = \mathbf{q}(u/2)$ . Express  $\mathbf{q}_1(u)$  as a degree three Bézier curve by giving its four control points.

9. [20 points] (Degree 3 Bézier curve.) A particle is animated so that it starts  $\langle 0, 0, 0 \rangle$  at time  $u = 0$  and arrives at  $\langle 4, 0, 2 \rangle$  at time  $u = 1$ . The initial velocity (at time  $u = 0$ ) is equal to  $\langle 6, 0, 0 \rangle$ , and its final velocity (at time  $u = 1$ ) is equal to  $\langle 0, 0, 3 \rangle$ . The motion is modeled with a degree three Bézier curve  $\mathbf{q}(u)$ .

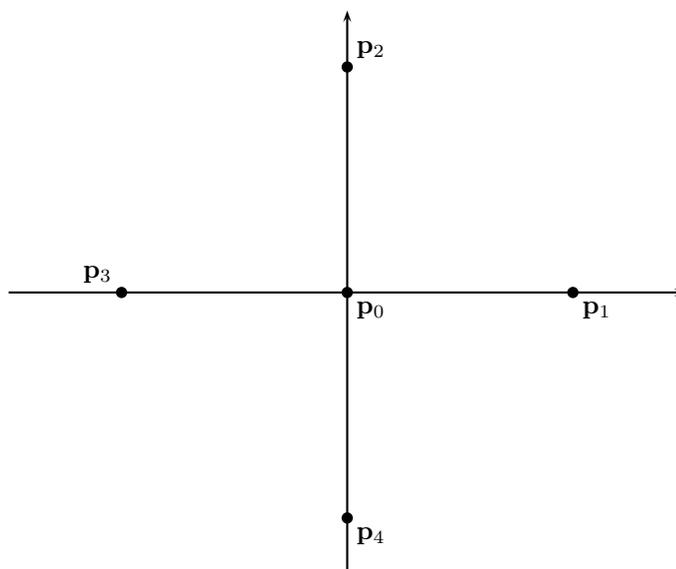
- Give the control points for the Bézier curve.
- Use the de Casteljau method to find the position of the particle at time  $u = \frac{1}{2}$ . (Show your work, including the values of  $\mathbf{r}_0, \mathbf{r}_1, \mathbf{r}_2, \mathbf{s}_0, \mathbf{s}_1, \mathbf{t}_0$ .)
- What is the velocity of the particle at time  $u = \frac{1}{2}$ ? First express this a general formula in terms of (some of)  $\mathbf{p}_0, \mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3, \mathbf{r}_0, \mathbf{r}_1, \mathbf{r}_2, \mathbf{s}_0, \mathbf{s}_1, \mathbf{t}_0$ . Then compute the velocity numerically. (THIS IS LITTLE TRICKY.)

10. [20 points] A Catmull-Rom curve is defined for the points  $\mathbf{p}_0 = \langle -1, 0 \rangle$ ,  $\mathbf{p}_1 = \langle 0, 0 \rangle$ ,  $\mathbf{p}_2 = \langle 0, 1 \rangle$ ,  $\mathbf{p}_3 = \langle 2, 1 \rangle$ ,  $\mathbf{p}_4 = \langle 2, 0 \rangle$ , and  $\mathbf{p}_5 = \langle 4, 0 \rangle$ ; so it interpolates  $\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3, \mathbf{p}_4$ . Compute the control points for all the Bézier segments of the Catmull-Rom curve. Graph and label the control points on the figure, draw the control polygons, and freehand sketch the Catmull-Rom curve. (Be sure to show interpolation points, and the slopes at interpolation points clearly!)



3. A Bézier curve  $\mathbf{q}(u)$  of degree three has control points  $\mathbf{p}_0 = \langle 0, 0 \rangle$ ,  $\mathbf{p}_1 = \langle 2, 0 \rangle$ ,  $\mathbf{p}_2 = \langle 2, 1 \rangle$ , and  $\mathbf{p}_3 = \langle 3, -1 \rangle$ . Give a scale handdrawing of the curve, showing its starting and ending points and starting and ending slopes clearly. Give the formula for  $\mathbf{q}(u)$ , as a cubic polynomial. Evaluate  $\mathbf{q}(\frac{1}{2})$  *using this formula*.
4. Let  $\tilde{q}(u)$  be the curve from the previous problem. Use the de Casteljau method to compute  $\mathbf{q}(\frac{1}{2})$  and  $\mathbf{q}(\frac{1}{4})$ . Draw a picture showing your work.
5. (Recursive subdivision) Work with the same curve  $\mathbf{q}(u)$  again. Consider the first half of the curve where  $u \in [0, \frac{1}{2}]$ . It is also a Bézier curve. What are the control points for this first half of  $\mathbf{q}(u)$ ?
7. Give the Catmull-Rom spline that is defined by the points  $\mathbf{p}_0 = \langle 0, 0 \rangle$ ,  $\mathbf{p}_1 = \langle 1, 0 \rangle$ ,  $\mathbf{p}_2 = \langle 1, 1 \rangle$ ,  $\mathbf{p}_3 = \langle 2, 1 \rangle$ , and  $\mathbf{p}_4 = \langle 3, 0 \rangle$ . How many cubic segments does it have? At what points does the curve start and end? What are the control points for its segments? Draw a picture showing the Catmull-Rom curve and its control points.

9. A degree three Bézier curve  $\mathbf{q}(u)$  has the four control points  $\mathbf{0} = \langle 0, 0 \rangle$ ,  $\mathbf{p}_1 = \langle 2, 0 \rangle$ ,  $\mathbf{p}_2 = \langle 0, 2 \rangle$ , and  $\mathbf{p}_3 = \langle 4, 4 \rangle$ .
- Sketch and label the control points and the control polygon for the Bézier curve.
  - Use the de Casteljau algorithm to compute the the point  $\mathbf{q}(\frac{1}{2})$ .
  - Sketch the Bézier curve approximately. **For both (a) and (c): Draw your sketch to scale as best you can and, when drawing the Bézier curve, show clearly the initial and final points and the initial and final slopes.**
10. [20 points] (Catmull-Rom interpolation.) Let  $\mathbf{p}_0 = \langle 0, 0 \rangle$ ,  $\mathbf{p}_1 = \langle 2, 0 \rangle$ ,  $\mathbf{p}_2 = \langle 0, 2 \rangle$ ,  $\mathbf{p}_3 = \langle -2, 0 \rangle$ , and  $\mathbf{p}_4 = \langle 0, -2 \rangle$ .
- Draw the Catmull-Rom curve that is defined by these points. Be sure to show clearly the starting point, ending point, and the slopes of the curve at each point  $\mathbf{p}_i$  on the curve.



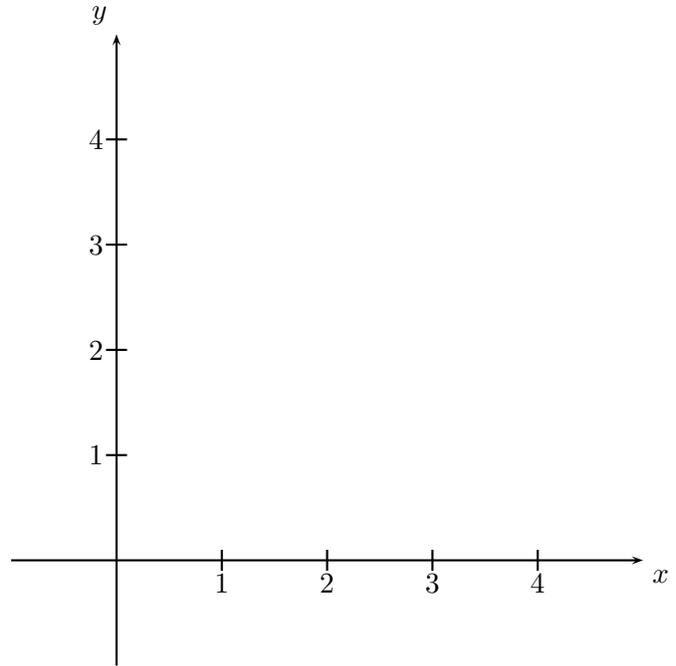
- What are the control points for the first Bézier segment of the Catmull-Rom curve above?

8.[20 points] A particle moving in  $\mathbb{R}^2$  is animated so that its position  $\mathbf{p}(u)$  at time  $u$  is given by a degree Bezier curve with control points  $\mathbf{p}_0, \mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3$ .

At time  $u = 0$ , the particle is at  $\langle 0, 0 \rangle$  and has velocity  $\langle 6, 0 \rangle$ .

At time  $u = 1$ , the particle is at  $\langle 0, 4 \rangle$  and has velocity  $\langle -12, 0 \rangle$ .

a. What are the control points  $\mathbf{p}_0, \mathbf{p}_1, \mathbf{p}_2,$  and  $\mathbf{p}_3$  equal to?



b. Draw on the graph, the four control points, and the control polygon.

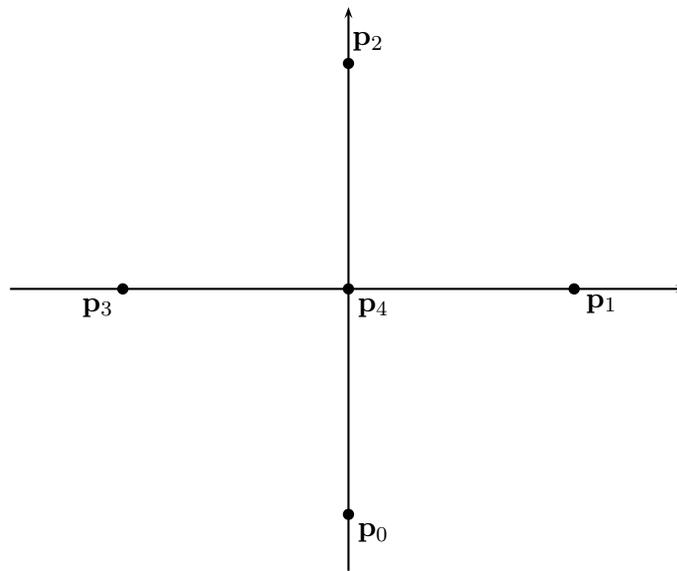
c. Use the de Casteljau algorithm to calculate  $\mathbf{p}(\frac{1}{2})$ , the position at time  $u = \frac{1}{2}$ . Show your work on the same graph.

d. Sketch the Bezier curve for  $0 \leq u \leq 1$  on the same graph. Be sure to show the starting and ending slopes (tangencies) clearly.

e. Consider the second half of the path of the particle, namely the path traced out when  $\frac{1}{2} \leq u \leq 1$ . This is also a degree three Bezier curve. What are its control points?

9. [20 points] (Catmull-Rom interpolation.) Let  $\mathbf{p}_0 = \langle 0, -2 \rangle$ ,  $\mathbf{p}_1 = \langle 2, 0 \rangle$ ,  $\mathbf{p}_2 = \langle 0, 2 \rangle$ ,  $\mathbf{p}_3 = \langle -2, 0 \rangle$ , and  $\mathbf{p}_4 = \langle 0, 0 \rangle$ ,

- (a) Draw the Catmull-Rom curve that is defined by these points. Be sure to show clearly the starting point, ending point, and the slopes of the curve at each point  $\mathbf{p}_i$  on the curve.



- (b) What are the control points for the first Bézier segment of the Catmull-Rom curve?