Name: Answer Key

PID:

1. Let \( x = (-2, 0, 4) \) and \( y = (4, 6, -2) \) be points in \( \mathbb{R}^3 \). Let \( u = (-4, 0, 8, 2) \) and \( v = (12, 18, -6, 3) \) be homogeneous representations of \( x \) and \( y \) (respectively). Find scalars \( \alpha \) and \( \beta \) so that \( w = \alpha u + \beta v \) is a homogeneous representation of the midpoint of \( x \) and \( y \). In other words, \( w \) is an affine combination of \( u \) and \( v \), and a homogeneous representation of \( \frac{1}{2}(x + y) \).

\[
\alpha = \frac{3}{5}, \quad \beta = \frac{2}{5}
\]

(To solve this, just pay attention to the weights 2 and 3 of \( \tilde{u} \) and \( \tilde{v} \).

\[\uparrow\ \text{"} \alpha \text{" means proportional to}\]

Need \( \alpha \approx \frac{1}{3}, \beta \approx \frac{1}{3} \) and \( \alpha + \beta = 1 \).

2. A triangle in \( \mathbb{R}^2 \) has three vertices \( x = (0, 0), y = (3, 3) \) and \( z = (6, 0) \). The point \( a = (3, 1) \) has barycentric coordinates \( \alpha = \frac{1}{3}, \beta = \frac{1}{3} \), and \( \gamma = \frac{1}{3} \).

Let \( u = (2x; 2), v = (y; 1) \) and \( w = (2z; 2) \) be homogeneous representations of \( x, y \) and \( z \), respectively.

Find values \( \alpha', \beta', \gamma' \) so that the affine combination \( \alpha' u + \beta' v + \gamma' w \) is a homogeneous representation of \( u \).

\[
\alpha' = \frac{1}{y}, \quad \beta' = \frac{2}{y}, \quad \gamma' = \frac{1}{y}
\]

This was easy to solve since \( \alpha' = \beta' = \gamma' = \frac{1}{3} \).

Need \( \alpha' = \frac{\alpha/\omega_x}{\alpha/\omega_x + \beta/\omega_y + \gamma/\omega_z} \) and similarly \( \beta', \gamma' \).

Where \( \omega_x, \omega_y, \omega_z \) are the weights of \( \omega_x, \omega_y, \omega_z \).

Alternatively: \( \alpha' \propto \alpha/\omega_x, \beta' \propto \beta/\omega_y, \gamma' \propto \gamma/\omega_z \).