1. Let $f$ be the linear transformation of $\mathbb{R}^2$ defined by

$$f((x,y)) = \begin{pmatrix} 2 & 0 \\ 1 & 3 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

Suppose $p = (0,3)$ and that $(2,-1)$ is normal to $C$ at $p$. Give a vector $m$ that is normal to the point $f(p)$ on the transformed curve $f(C)$. (It does not need to be a unit vector.)

$$\begin{pmatrix} \frac{1}{2} \\ -\frac{1}{6} \end{pmatrix}^T = \begin{pmatrix} \frac{1}{2} & -\frac{1}{6} \\ 0 & \frac{1}{3} \end{pmatrix}$$

$$\vec{m} = \begin{pmatrix} \frac{1}{2} \\ -\frac{1}{6} \end{pmatrix} \begin{pmatrix} 2 \\ -1 \end{pmatrix} = \begin{pmatrix} \frac{7}{6} \\ -\frac{1}{3} \end{pmatrix}$$

2. Now let $g$ be the affine transformation of $\mathbb{R}^2$ defined by

$$f((x,y)) = \begin{pmatrix} 2 & 0 \\ 1 & 3 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} 0 \\ 2 \end{pmatrix}$$

(So $g$ is the composition of $f$ and a translation.) Again suppose $p = (0,3)$ lies on a curve $C$ and that $(2,-1)$ is normal to $C$ at $p$. Give a vector $m$ that is normal to the point $g(p)$ on the transformed curve $f(C)$.

Translations do not affect normals

$$\vec{m} = \begin{pmatrix} \frac{7}{6} \\ -\frac{1}{3} \end{pmatrix}$$