Matrix representations

In ordinary coordinates $\langle x, y \rangle \in \mathbb{R}^2$, or $\langle x, y, z \rangle \in \mathbb{R}^3$

2x2 or 3x3 matrices for linear maps

With homogeneous coordinates

3x3 or 4x4 matrices for affine maps

perspective maps, and "rational" map.

To form matrix:

Look at image of $\vec{O}$ $\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

" " " $\vec{i}$ $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$

" " " $\vec{j}$ $\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$

" " " $\vec{k}$ $\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

} for affine maps (not perspective)

Affine

$$\begin{pmatrix} u_1 & v_1 & w_1 & t_1 \\ u_2 & v_2 & w_2 & t_2 \\ u_3 & v_3 & w_3 & t_3 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$\vec{t}$ = image of $\vec{0}$ $\quad \vec{f}(\vec{0})$

$\vec{u} = \vec{f}(\vec{i}) - \vec{f}(\vec{0})$

$\vec{v} = \vec{f}(\vec{j}) - \vec{f}(0)$

$\vec{w} = \vec{f}(\vec{k}) - \vec{f}(0).$

Rational maps

$$\langle \vec{x}, \vec{y}, \vec{z} \rangle \mapsto \left\langle \frac{p(x,y,z)}{w(x,y,z)}, \frac{q(x,y,z)}{w(x,y,z)}, \frac{r(x,y,z)}{w(x,y,z)} \right\rangle$$

p, q, r, w - degree 1 polynomials

Example:

$$\langle x, y, z \rangle \mapsto \left\langle \frac{3x+1}{x+y}, \frac{3z+1}{x+y}, \frac{x}{x+y} \right\rangle$$

$$\langle x, y, z, 1 \rangle \mapsto \langle \quad , \quad , \quad , 1 \rangle$$

$$\langle x, y, z, 1 \rangle \mapsto \langle 3x+1,\ 3z+1,\ x,\ x+y \rangle$$

$$\left\langle \frac{x}{w}, \frac{y}{w}, \frac{z}{w}, 1 \right\rangle \mapsto \left\langle 3\frac{x}{w}+1,\ 3\frac{z}{w}+1,\ \frac{x}{w},\ \frac{x}{w}+\frac{y}{w} \right\rangle$$

$$\langle x, y, z, w \rangle \mapsto \langle 3x+w,\ 3z+w,\ x,\ x+y \rangle$$

$$\begin{pmatrix} 3x+w \\ 3z+w \\ x \\ x+y \end{pmatrix} = \begin{pmatrix} 3 & 0 & 0 & 1 \\ 0 & 0 & 3 & 1 \\ 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix}$$

Let $\vec{u} = \langle 1, 2, 3 \rangle$

Let $f(\vec{x}) = \vec{u} \times \vec{x}$

Express f as a 3x3 matrix

$$M = \begin{pmatrix} 0 & -3 & 2 \\ 3 & 0 & -1 \\ -2 & 1 & 0 \end{pmatrix}$$

Linear
$$\begin{cases} f(\vec{0}) = \vec{0} \\ f(\vec{x} + \vec{y}) = f(\vec{x}) + f(\vec{y}) \\ f(\alpha \vec{x}) = \alpha f(\vec{x}) \end{cases}$$

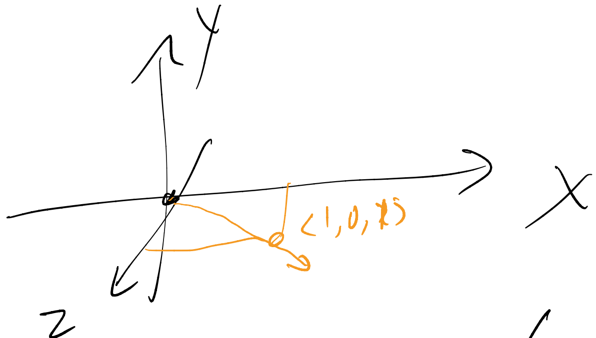
$\vec{u} \times \vec{i} = \langle 1, 2, 3 \rangle \times \langle 1, 0, 0 \rangle$
$= \langle 0, 3, -2 \rangle$

$\vec{u} \times \vec{j} = \langle 1, 2, 3 \rangle \times \langle 0, 1, 0 \rangle$

$\vec{u} \times \vec{k} = \langle 1, 2, 3 \rangle \times \langle 0, 0, 1 \rangle$

Quiz 5

$R_{\pi, \langle 1,0,1\rangle}$ or $R_{180^0, \langle 1, 0, 1\rangle}$



$\vec{i} \mapsto \vec{k}$

$\vec{j} \mapsto -\vec{j}$

$\vec{k} \mapsto \vec{i}$

$$\begin{pmatrix} 0 & 0 & 1 \\ 0 & -1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

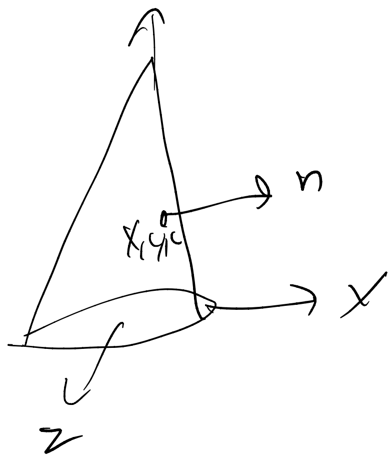
Normals & Directions:

A function $y = f(x,y)$ - defines a surface

, normal is $\langle n_x, n_y, n_z \rangle$

Normals upward..

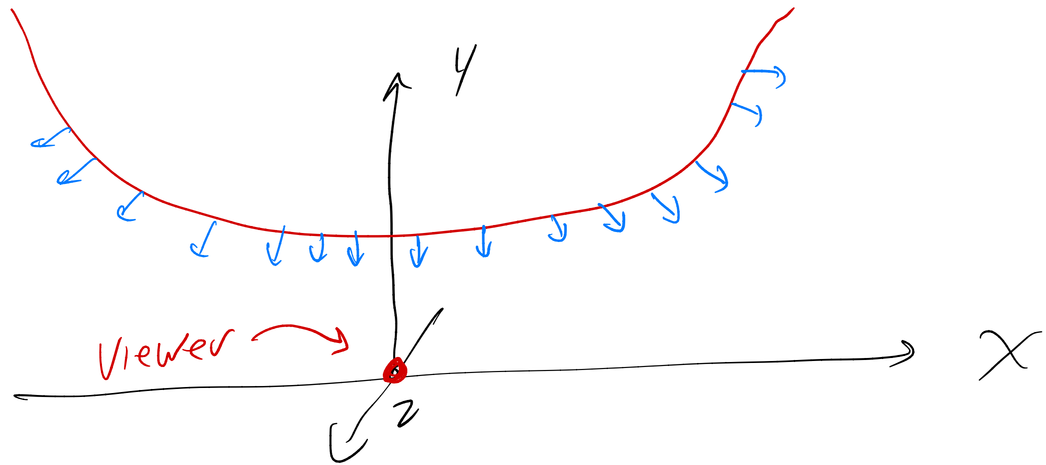
y - component: $n_y > 0$



n_x, n_z - in same direction of x, z of cone

is centered on y-axis

Here also $n_y > 0$



y
Viewer
x
z

Phong lighting:

Calculation of specular, diffuse, ambient as a function of $\bar{l}, \vec{v}, \vec{n}$ & material properties & light properties

Can be used with either

- Phong interpolation (shading) ← Phong lighting at every pixel

or

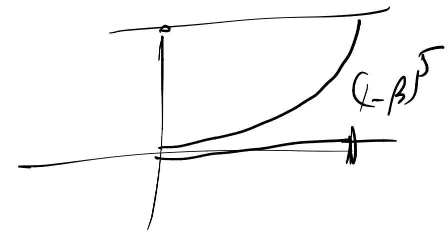
- Gouraud interpolation (shading) ← Phong lighting at ~~each~~ every vertex

Schlick Fresnel specular adjustment

Instead of ρ_s,

use $\text{lerp}(\rho_s, 1, (1-\beta)^5)$

~~$\beta = \cos(\vec{l} \cdot \vec{n})$~~ β -



$$A(\vec{x}) = R_\theta \vec{x} + \vec{t}$$

$$\vec{y} = R_\theta \vec{x} + \vec{t}$$ Solve for $\vec{x} = y$

$$\vec{x} = R_\theta \vec{x} + \vec{t}$$

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