Recall Phong lighting [Phong 1975]

Ambient, Diffuse, Specular, Emissive - light component

Material Properties $\rho_a, \rho_d, \rho_s, f, I_e$

Light properties $I_a^{in}, I_d^{in}, I_s^{in}$

Distance attenuation:

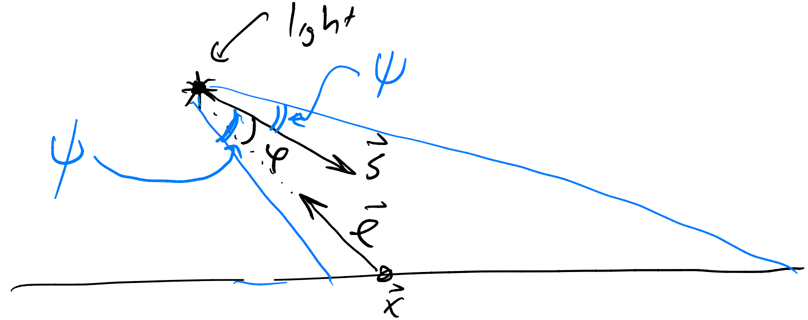
Multiplicative factor δ if $\delta = 1$ no attenuation

$$\delta = \frac{1}{k_c + k_\ell d + k_q \cdot d^2}$$

where d = distance from the light position to the point x being illuminated.

δ is applied to ambient, diffuse & specular components.

Spotlight:



$\vec{s}$ - unit vector
Spot direction

$\vec{\ell}$ - unit vector ~~to~~wards the light

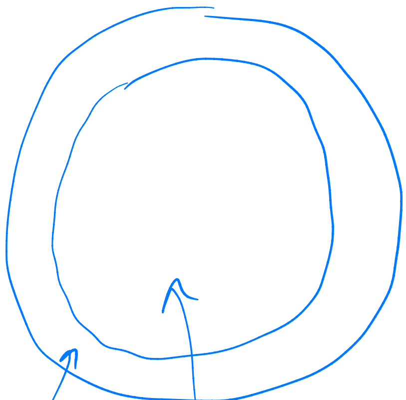
φ - angle between $\vec{\ell}$ and $\vec{s}$. $\cos\varphi = -\vec{\ell}\cdot\vec{s}$

ψ - spot cutoff angle. Cone of spotlight stops at angle ψ from central axis $\vec{s}$

c - spot exponent $c=0$ for no attenuation ~~to~~ towards the side of the spot cone
$c>0$ - spotlight diminishes with φ

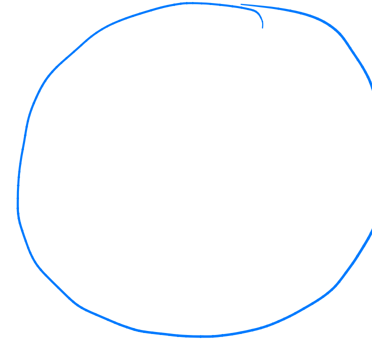
"spotlight"

$$\delta^{sp} = \begin{cases} 0 & \text{if } \varphi > \psi, \text{ i.e. } -\ell\cdot s < \cos\psi \\ \underbrace{(-\vec{\ell}\cdot\vec{s})^c}_{(\cos\varphi)^c} & \text{if } \varphi \le \psi, \text{ i.e. } (-\ell\cdot s) \ge \cos\psi \end{cases}$$



full brightness

diminishing brightness here



$c = 0$ – constant brightness

$c > 0$ brightness fades towards the edge

Not included in the traditional formulas

Spotlight light value δ^{sp} is a multiplicative factor applied to diffuse & specular – not usually to ambient.

Putting it all together: Let λ be a color (R, G, B)

$$I^\lambda = I_a^\lambda + I_d^\lambda + I_s^\lambda + I_e^\lambda$$

outgoing light

$$= I_e^\lambda + \rho_a^\lambda I_a^{in,\lambda,global} \quad \leftarrow \text{ambient}$$

$$+ \sum_{lights\ i} \delta_i \rho_a^\lambda I_a^{in,\lambda,i}$$

δ_i – depends on position of i-th light

$$+ \sum_{light\ i} \delta_i \delta_i^{sp} \rho_d^\lambda I_d^{in,\lambda,i} (\vec{l}_i \cdot \vec{n}) \quad \leftarrow \text{diffuse}$$

$$+ \sum_{light\ i} \delta_i \delta_i^{sp} \rho_s^\lambda I_s^{in,\lambda,i} (\vec{h}_i \cdot \vec{n})^f \quad \leftarrow \text{specular}$$

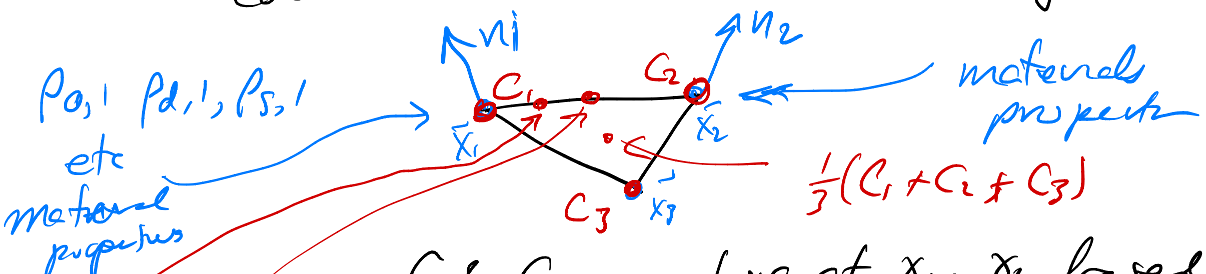
or Use $\vec{\rho}_{schlick}$ instead of ρ_s – $\vec{\rho}_{schlick} = (1-\beta)\vec{\rho}_s + \beta \qquad \beta = (1 - \vec{e}\cdot\vec{n})^5$

or Instead of $\vec{h}_i \cdot \vec{n}$ use $\max\{0, \vec{r}_i \cdot \vec{V}\}$ $\quad \vec{r}_i$ – reflection vector.

Phong interpolation & Gouraud interpolation "shading"

aka Phong shading / Gouraud shading.

Gouraud interpolation - lighting calculation carried out each vertex - colours are averaged across the triangle.

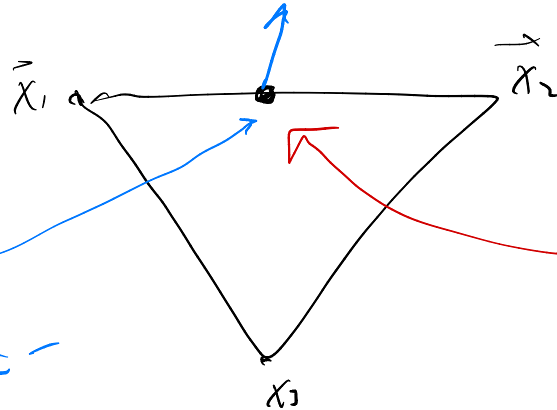


C_1 & C_2 - colours at x_1, x_2 based on Phong lighting calculation

$\frac{3}{4}C_1 + \frac{1}{4}C_2$

$\frac{1}{2}(C_1 + C_2)$

Phong interpolation: Average material properties, normals and position $\vec{x}$ across the triangle to each pixel → Do the Phong lighting calculation for each pixel.



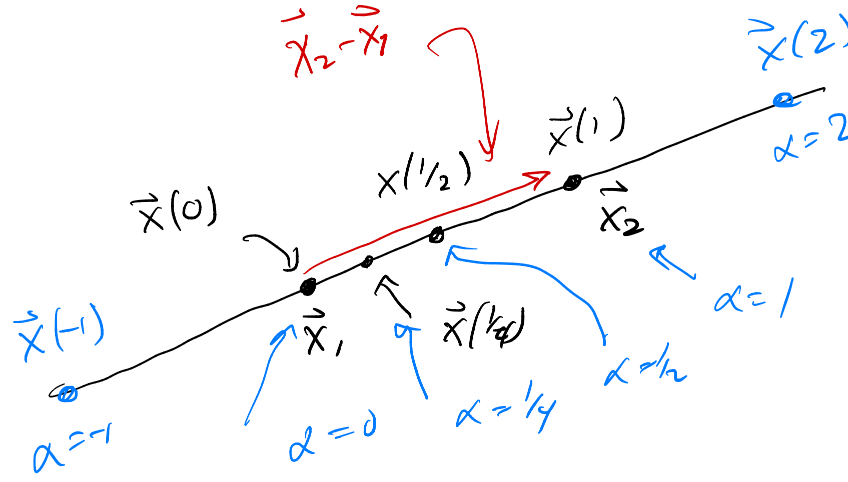
normal & material properties & vertex position — calculated by averaging

Do Phong lighting based on the averaged (interpolated) values.

Interpolation:

Linear interpolation:

Define $\vec{u} = \vec{x}(\alpha)$ to be fraction α of the way from $\vec{x}_1$ to $\vec{x}_2$



$\vec{x}_1, \vec{x}_2 \in \mathbb{R}^k$

$k \geq 1$

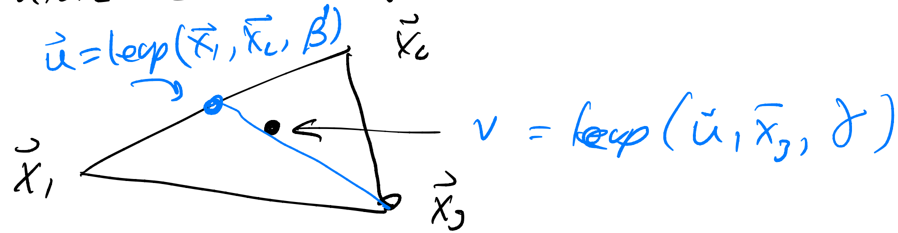
Formula $\vec{x}(\alpha) = (1-\alpha)\,\vec{x}_1 + \alpha\,\vec{x}_2 = \vec{x}_1 + \alpha(\vec{x}_2 - \vec{x}_1)$

$= \text{lerp}(\vec{x}_1, \vec{x}_2, \alpha)$ aka $\text{mix}(\vec{x}_1, \vec{x}_2, \alpha)$.

What is $\vec{x}(2)$ $(\alpha = 2)$ equal to?

For $\alpha < 0$ or $\alpha > 1$, we also call it linear extrapolation.

Look ahead to barycentric coordinates



Use linear interpolation to estimate function values:

We know $f(\vec{x}_1) = c_1$ & $f(\vec{x}_2) = c_2$ (say)

For $\vec{u} = x(\alpha) = \text{lerp}(\vec{x}_1, \vec{x}_2, \alpha)$

Estimate $f(\vec{u}) = \text{lerp}(f(\vec{x}_1), f(\vec{x}_2), \alpha) = \text{lerp}(c_1, c_2, \alpha)$

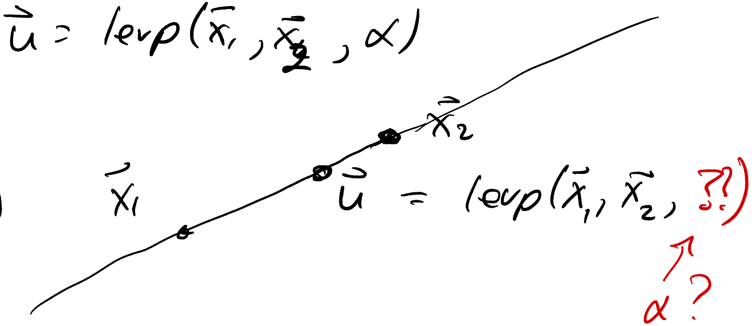
Same formula can be used for linear extrapolation.

Inverting linear interpolation

Given $\vec{x}_1$, $\vec{x}_2$, $\vec{u}$.

Want to find α s.t. $\vec{u} = lerp(\vec{x}_1, \vec{x}_2, \alpha)$

Assume $\vec{u}$ is on the line



Know: $\vec{u} = (1-\alpha)\vec{x}_1 + \vec{x}_2 = \vec{x}_1 + \alpha(\vec{x}_2 - \vec{x}_1)$

So $\vec{u} - \vec{x}_1 = \alpha(\vec{x}_2 - \vec{x}_1)$

Take dot product with $\vec{x}_2 - \vec{x}_1$

$$(\vec{u}-\vec{x}_1)\cdot(\vec{x}_2-\vec{x}_1) = \alpha(\vec{x}_2-\vec{x}_1)\cdot(\vec{x}_2-\vec{x}_1) = (\vec{x}_2-\vec{x}_1)^2 = \underbrace{\|\vec{x}_2-\vec{x}_1\|^2}_{\text{nonzero}}$$

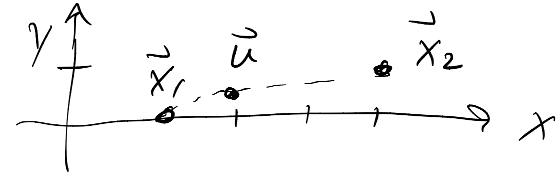
Then

$$\boxed{\alpha = \frac{(\vec{u}-\vec{x}_1)(\vec{x}_2-\vec{x}_1)}{(\vec{x}_2-\vec{x}_1)^2}}$$

$\alpha = 3$

Example $\vec{x}_1 = \langle 1, 0\rangle$, $\vec{x}_2 = \langle 4, 1\rangle$

$\vec{u} = \langle 2, 1/3\rangle$



$\vec{u} - \vec{x}_1 = \langle 1, 1/3\rangle$

$\vec{x}_2 - \vec{x}_1 = \langle 3, 1\rangle$ $\qquad (\vec{x}_2 - \vec{x}_1)^2 = 3^2 + 1^2 = 10$ (Denominator)

$(\vec{u} - \vec{x}_1)\cdot(\vec{x}_2 - \vec{x}_1) = \langle 1, 1/3\rangle \cdot \langle 3, 1\rangle = 3 + 1/3 = 10/3$

$\alpha = \dfrac{10/3}{10} = 1/3$ $\boxed{\checkmark}$

Why not just components:

① Avoid division by zero, or division by near-zero.

② Nice formula - avoids division by small numbers + comparisons

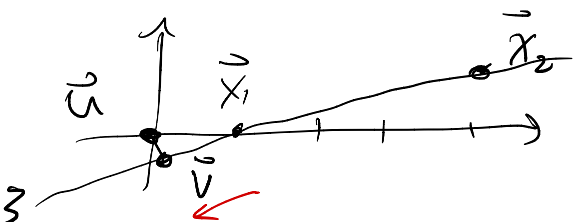
And It is robust if $\vec{u}$ is not on the line.

For $\vec{u}$ not on the line

$$(\vec{u}-\vec{x}_1)\cdot(\vec{x}_2-\vec{x}_1) = (\vec{v}-\vec{x}_1)\cdot(\vec{x}_2-\vec{x}_1)$$

Let $\vec{v}$ be the point on the line closest to $\vec{u}$

Example Line $\vec{x}_1 = \langle 1,0\rangle$, $\vec{x}_2 = \langle 4,1\rangle$ $\vec{u} = \langle 0,0\rangle = \vec{0}$



$$\vec{u}-\vec{x}_1 = \langle -1,0\rangle$$

$$(\vec{u}-\vec{x}_1)\cdot(\vec{x}_2-\vec{x}_1) = \langle -1,0\rangle\cdot\langle 3,1\rangle = -3$$

$$(\vec{x}_2-\vec{x}_1)^2 = 10$$

$$\alpha = \frac{-3}{10}$$

$$\vec{v} = x\left(\frac{-3}{10}\right) = \left(1-\frac{-3}{10}\right)\vec{x}_1 + \frac{-3}{10}\vec{x}_2$$

$$= \frac{13}{10}\langle 1,0\rangle - \frac{3}{10}\langle 4,1\rangle = \left\langle \frac{1}{10}, \frac{-3}{10}\right\rangle$$

- Closest point on the line to $\vec{u}$ (the origin)