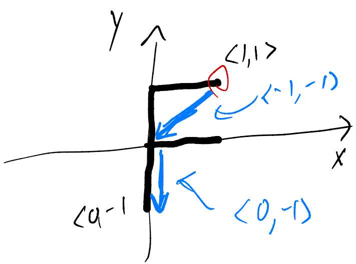
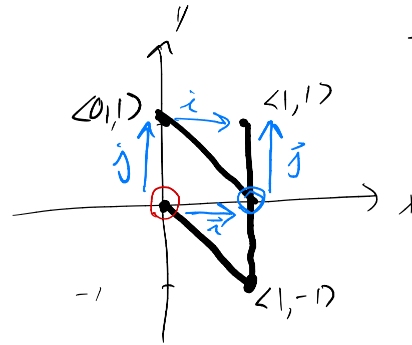
From Piazza

$$f(x) = \begin{pmatrix} -1 & 0 \\ 1 & -1 \end{pmatrix} \vec{x} + \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$



f

$\Rightarrow$



$f(\vec{x}) = M\vec{x} + \vec{u}$

To find $f^{-1}(x) = N\vec{x} + \vec{v}$

Let $\vec{y} = M\vec{x} + \vec{u}$, Solve for $\vec{x}$

$\vec{y} - \vec{u} = M\vec{x}$

$\vec{x} = M^{-1}y - M^{-1}\vec{u}$

N is M^{-1} and $\vec{v} = -M^{-1}\vec{u}$

How to compute f^{-1}

$$f^{-1}(\vec{0}) = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

Consider the linear part $\longrightarrow$ maps to $\swarrow$ under f^{-1}

$\uparrow$ " " $\downarrow$ " f^{-1}

$$f^{-1}(\vec{x}) = \begin{pmatrix} -1 & 0 \\ -1 & -1 \end{pmatrix} \vec{x} + \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

Homogeneous Coordinates (in $\mathbb{R}^2$)

Defn For $w \neq 0$, $\langle x, y, w \rangle = \begin{pmatrix} x \\ y \\ w \end{pmatrix}$ is homogeneous coordinates for $\langle x/w, y/w \rangle$ in $\mathbb{R}^2$.

Example $\langle -2, 1 \rangle$ has homogeneous coordinates

$$\langle -2, 1, 1 \rangle, \quad \langle -4, 2, 2 \rangle, \quad \langle -1, 1/2, 1/2 \rangle,$$

$$\langle -6, 3, 3 \rangle, \quad \langle 4, -2, -2 \rangle, \quad \langle 2, -1, -1 \rangle, \ldots$$

Representing an affine transformation in $\mathbb{R}^2$ with a 3x3 matrix over homogeneous coordinates

Let $A(\vec{x}) = B(\vec{x}) + \vec{a} = M\vec{x} + \vec{u}$ $\quad M$ is 2×2 $\quad B$ - linear

$$= \begin{pmatrix} a & b \\ c & d \end{pmatrix} \vec{x} + \begin{pmatrix} e \\ f \end{pmatrix}$$

Let $N = \begin{pmatrix} a & b & e \\ c & d & f \\ 0 & 0 & 1 \end{pmatrix}$

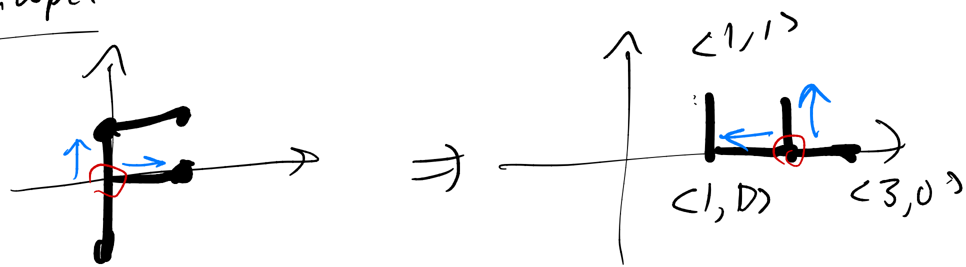
<u>Thm</u> If $w \neq 0$, $N\begin{pmatrix} x \\ y \\ w \end{pmatrix}$ is homogeneous coordinates for $A(\langle \frac{x}{w}, \frac{y}{w} \rangle)$.

Pf: $N\begin{pmatrix} x \\ y \\ 1 \end{pmatrix} = \begin{pmatrix} ax+by+e \\ cx+dy+f \\ 1 \end{pmatrix}$ which represents $\langle ax+by+e, cx+dy+f \rangle$
$= A(\langle x, y \rangle)$.

$$N\begin{pmatrix} wx \\ wy \\ w \end{pmatrix} = \begin{pmatrix} w(ax+by+e) \\ w(cx+dy+f) \\ w \end{pmatrix}$$ also represents $A(\langle x,y\rangle)$

In fact, for any $\alpha \neq 0$, αN represents A over homogeneous coordinates.

Example

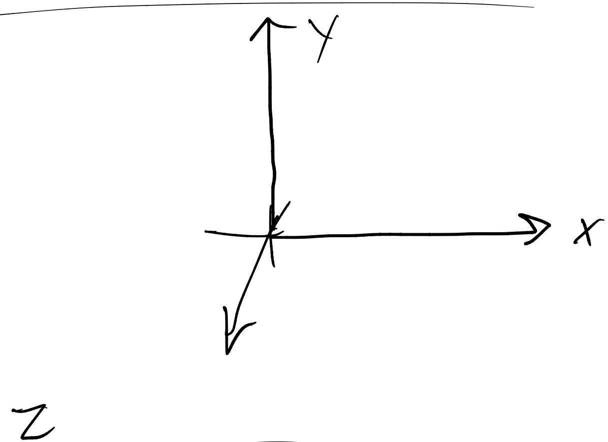


$$N = \begin{pmatrix} 0 & -1 & 2 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$\mathbb{R}^3$

$$\langle x, y, z \rangle = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \in \mathbb{R}^3$$

Conventions on axes

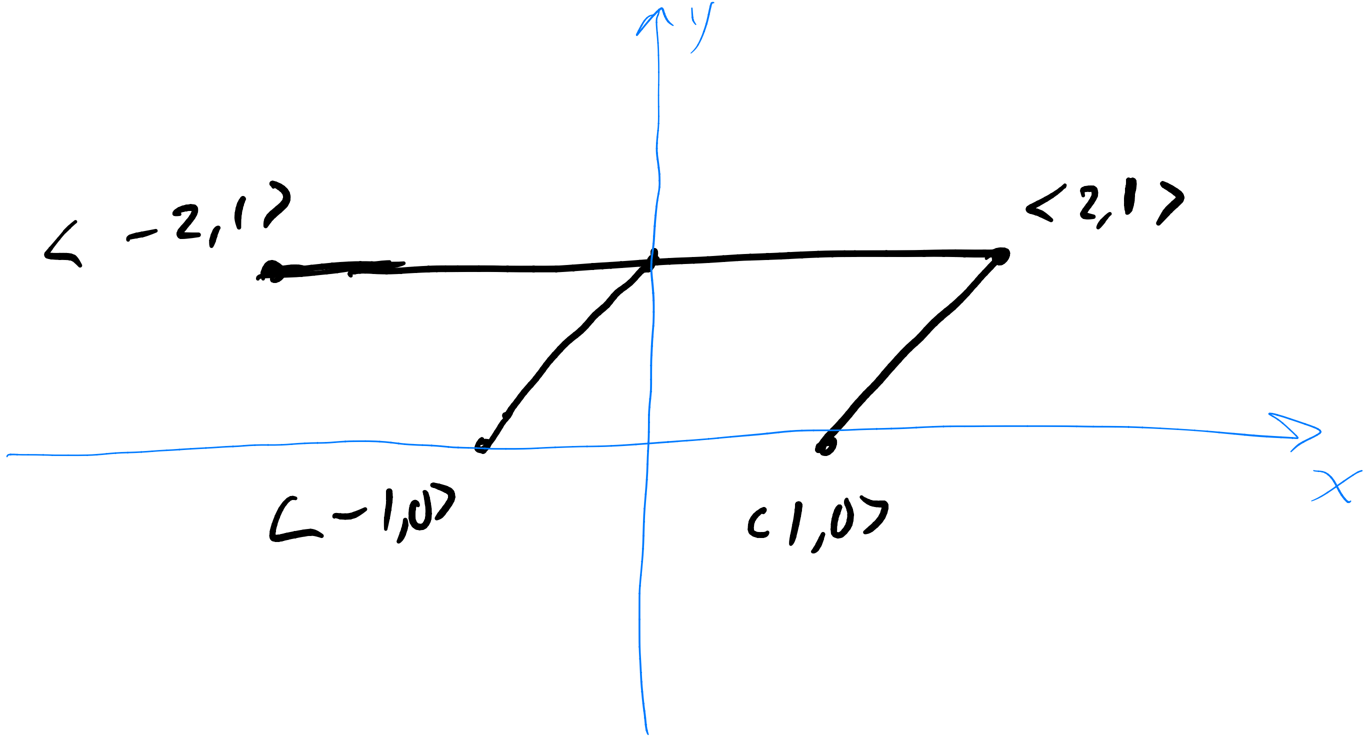


y - upward

x - rightward

z - towards viewer

Important!


y
<-2,1>
<2,1>
x
<-1,0>
<1,0>