Math 155A — Computer Graphics — Winter 2020
Homework #7 — Due Monday, March 16, 4:00pm
Hand in via Gradescope — Use separate pages for each problem.

1. a. What subtractive colors should be combined to the form the color *Red*?

b. What subtractive colors should be combined to the form the color *Black*?

c. What additive colors should be combined to the form the color *Magenta*?

d. What additive colors should be combined to the form the color *White*?

2. A Bézier curve $\mathbf{q}(u)$ in $\mathbb{R}^2$ has control points $\mathbf{p}_0 = \langle -1, 1\rangle$, $\mathbf{p}_1 = \langle 0, 0\rangle$, $\mathbf{p}_2 = \langle 2, 2\rangle$ and $\mathbf{p}_3 = \langle 2, 0\rangle$. What are the values of $\mathbf{q}(0)$ and $\mathbf{q}(1)$ and of the derivatives $\mathbf{q}'(0)$ and $\mathbf{q}'(1)$? Draw a graph showing the control points, the control polygon, and the approximate curve. Be sure to show the starting and end points and starting and end tangencies clearly.

3. Let $\mathbf{q}(u)$ be the same degree Bézier curve as in problem 1.

a. Evaluate $\mathbf{q}(\frac{2}{3})$ using the de Casteljau algorithm. Draw a graph showing your work and all intermediate values.

b. The first two-thirds of $\mathbf{q}(u)$ is a Bézier curve joining the points $\mathbf{q}(0)$ and $\mathbf{q}(\frac{2}{3})$. What are its control points?

4. A Catmull-Rom curve is defined with control points $\mathbf{p}_0 = \langle 0, 0\rangle$, $\mathbf{p}_1 = \langle 1, 1\rangle$, $\mathbf{p}_2 = \langle 2, 0\rangle$, $\mathbf{p}_3 = \langle 5, 1\rangle$, and $\mathbf{p}_4 = \langle 6, 2\rangle$. The Catmull-Rom curve is a C^1-continuous ***piecewise*** degree three Bézier curve defined on $[0, 2]$ with $\mathbf{q}(0) = \mathbf{p}_1$ and $\mathbf{q}(1) = \mathbf{p}_2$ and $\mathbf{q}(2) = \mathbf{p}_3$. The Catmull-Rom formula gives $\mathbf{q}'(0) = \langle 1, 0\rangle$ and $\mathbf{q}'(1) = \langle 2, 0\rangle$ and $\mathbf{q}'(2) = \langle 2, 1\rangle$.

a. What are the control points for the first "piece" of $\mathbf{q}$, joining $\mathbf{p}_1$ and $\mathbf{p}_2$?

b. What are the control points for the first "piece" of $\mathbf{q}$, joining $\mathbf{p}_2$ and $\mathbf{p}_3$?

5. Consider the situation where $\mathbf{q}(u)$ and $\mathbf{r}(u)$ are two degree Bézier curves, and where $\mathbf{q}(u)$ ends at $\langle 0, 0\rangle$ and $\mathbf{r}(u)$ begins at $\langle 0, 0\rangle$. They are joined together to make a piecewise degree three Bézier curve (with two "pieces").

a. Give an example of $\mathbf{q}(u)$ and $\mathbf{r}(u)$ so that the piecewise Bézier curve is G^1-continuous, but not C^1-continuous. Define $\mathbf{q}(u)$ and $\mathbf{r}(u)$ by giving their control points.

b. Give an example of $\mathbf{q}(u)$ and $\mathbf{r}(u)$ so that the piecewise Bézier curve is C^1-continuous, but not G^1-continuous. Define $\mathbf{q}(u)$ and $\mathbf{r}(u)$ by giving their control points. [Hint: The derivative can be zero; the control points may not all be distinct.]