Math 155A — Computer Graphics — Winter 2020
Homework #5 — Due Monday, March 2, 5:30pm
Hand in via Gradescope — Use separate pages for each problem.

NOTE: The Homowork Assignment now has SIX (6) questions (see second page).

1. Let $\mathbf{x}_1 = \langle -2, 0\rangle$ and $\mathbf{x}_2 = \langle 4, 1\rangle$. Let α control the linear interpolation (and linear extrapolation) from $\mathbf{x}_1$ to $\mathbf{x}_2$ by $\text{Lerp}(\mathbf{x}_1, \mathbf{x}_2, \alpha)$.

(a) What points are obtained with α equal to -2, -1, 0, $\frac{1}{10}$, $\frac{1}{3}$, $\frac{1}{2}$, 1, $1\frac{1}{2}$ and 2? What value of α gives the point $\langle 2, \frac{2}{3}\rangle$? The point $\langle 16, 3\rangle$? Graph your answers.
(b) What point $\mathbf{u}$ on the line containin $\mathbf{x}_1$ and $\mathbf{x}_2$ is the closest to the origin? Find the value α such that $\mathbf{u} = \text{Lerp}(\mathbf{x}_1, \mathbf{x}_2, \alpha)$.
(c) Suppose the values for $f(\mathbf{x}_1) = -3$ and $f(\mathbf{x}_2) = 3$ have been set, and we wish to set other values for $f(\mathbf{z})$ by linear interpolation/extrapolation. What will this set $f(\langle 2, \frac{2}{3}\rangle)$ equal to?

2. Let $\mathbf{u} = \text{Lerp}(\mathbf{x}, \mathbf{y}, \alpha)$.

(a) For what values of α must $\mathbf{u}$ be a linear combination of $\mathbf{x}$ and $\mathbf{y}$?
(b) For what values of α must $\mathbf{u}$ be an affine combination of $\mathbf{x}$ and $\mathbf{y}$?
(c) For what values of α must $\mathbf{u}$ be a weighted average of $\mathbf{x}$ and $\mathbf{y}$?

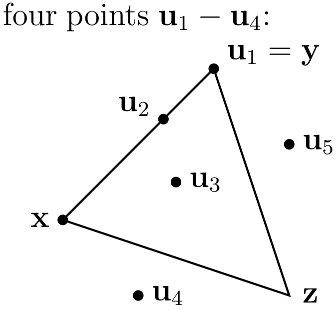
3. Let $\mathbf{x} = \langle 0, 1\rangle$, $\mathbf{y} = \langle 2, 3\rangle$, and $\mathbf{z} = \langle 3, 0\rangle$ in $\mathbb{R}^2$. Determine the points represented by the following sets of barycentric coordinates.

a. $\alpha = 0$, $\beta = 1$, $\gamma = 0$.
b. $\alpha = \frac{2}{3}$, $\beta = \frac{1}{3}$, $\gamma = 0$.
c. $\alpha = \frac{1}{3}$, $\beta = \frac{1}{3}$, $\gamma = \frac{1}{3}$.
d. $\alpha = \frac{4}{5}$, $\beta = \frac{1}{10}$, $\gamma = \frac{1}{10}$.
e. $\alpha = \frac{4}{3}$, $\beta = \frac{2}{3}$, $\gamma = -1$.

Graph your answers along with the triangle formed by $\mathbf{x}$, $\mathbf{y}$, and $\mathbf{z}$.

4. Let, again, $\mathbf{x} = \langle 0, 1\rangle$, $\mathbf{y} = \langle 2, 3\rangle$, and $\mathbf{z} = \langle 3, 0\rangle$. Determine the barycentric coordinates of the following points four points $\mathbf{u}_1 - \mathbf{u}_4$:

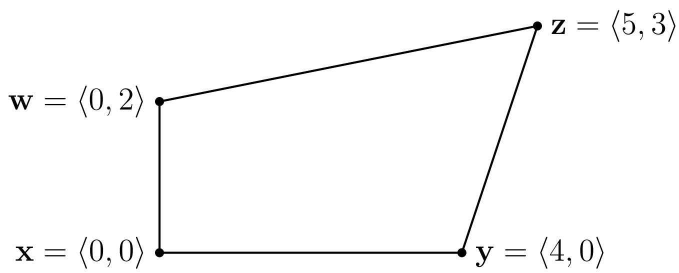
a. $\mathbf{u}_1 = \langle 2, 3\rangle$.
b. $\mathbf{u}_2 = \langle 1\frac{1}{3}, 2\frac{1}{3}\rangle$.
c. $\mathbf{u}_3 = \langle \frac{3}{2}, \frac{3}{2}\rangle$.
d. $\mathbf{u}_4 = \langle 1, 0\rangle$.



The figure also show a point labelled $\mathbf{u}_5$. For the barycentric coordinates for $\mathbf{u}_5$: Which of α, β, γ are positive? Which ones are negative? Which ones are zero?

5. Let $\mathbf{x} = \langle 0, 0\rangle$, $\mathbf{y} = \langle 4, 0\rangle$, $\mathbf{z} = \langle 5, 3\rangle$, and $\mathbf{w} = \langle 0, 2\rangle$, as shown in the figure. For each of the following values of α and β, what point is obtained by bilinear interpolation? (Give the coordinates of the points a.-d.) Then draw a copy of the quadrilaterial, and show the approximate locations of your four answers. (The value α gives the left-to-right direction; β the bottom-to-top direction.)

a. $\alpha = 0$ and $\beta = 1$.
b. $\alpha = \frac{2}{3}$ and $\beta = 1$.
c. $\alpha = \frac{1}{2}$ and $\beta = \frac{3}{4}$.
d. $\alpha = \frac{1}{3}$ and $\beta = \frac{2}{3}$.



6. Suppose a surface patch $\mathbf{u}(\alpha, \beta)$ in $\mathbb{R}^3$ is defined by bilinearly interpolating from four vertices. Derive the following formulas for the partial derivatives of $\mathbf{u}$:

$$\frac{\partial \mathbf{u}}{\partial \alpha} = (1-\beta)(\mathbf{y}-\mathbf{x}) + \beta(\mathbf{z}-\mathbf{w}) = \text{Lerp}(\mathbf{y}-\mathbf{x}, \mathbf{z}-\mathbf{w}, \beta)$$

$$\frac{\partial \mathbf{u}}{\partial \beta} = (1-\alpha)(\mathbf{w}-\mathbf{x}) + \alpha(\mathbf{z}-\mathbf{y}) = \text{Lerp}(\mathbf{w}-\mathbf{x}, \mathbf{z}-\mathbf{y}, \alpha).$$

In addition, give the general formula for a normal vector to the patch at a point $\mathbf{u} = \mathbf{u}(\alpha, \beta)$. (It does **not** need to be a unit vector.)