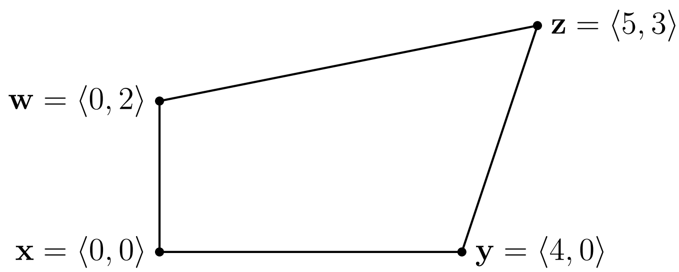
Math 155A — Computer Graphics — Winter 2019
Homework #5 — Due Tuesday, March 5, 4:00pm
Hand in via Gradescope — Use separate pages for each problem.

1. Why is it customary to use the same specular exponent for all wavelengths? What might a specular highlight look like if different wavelengths had different specular exponents?

2. Let $\mathbf{x} = \langle 0, 0\rangle$, $\mathbf{y} = \langle 4, 0\rangle$, $\mathbf{z} = \langle 5, 3\rangle$, and $\mathbf{w} = \langle 0, 2\rangle$, as shown in the figure. For each of the following values of α and β, what point is obtained by bilinear interpolation? Draw a copy of the quadrilateral, and show the approximate locations of your answers. (The value α gives the right-to-left direction; β the bottom-to-top direction.)

a. $\alpha = 1$ and $\beta = 0$.
b. $\alpha = \frac{1}{3}$ and $\beta = 1$.
c. $\alpha = \frac{1}{2}$ and $\beta = \frac{1}{4}$.
d. $\alpha = \frac{2}{3}$ and $\beta = \frac{1}{3}$.



3. A function $g : [0, 1] \times [0, 1] \to \mathbb{R}^3$ is defined by setting $g(\langle 0, 0\rangle) = \langle 2, 0, 0\rangle$, $g(\langle 1, 0\rangle) = \langle 0, 1, 0\rangle$, $g(\langle 0, 1\rangle) = \langle 0, 2, 4\rangle$, $g(\langle 1, 1\rangle) = \langle 2, 2, 0\rangle$, and then using bilinear interpolation to extend the domain of g to the square $[0, 1] \times [0, 1]$. What is $g(\frac{1}{4}, 0)$? $g(\frac{1}{4}, 1)$? $g(\frac{1}{4}, \frac{1}{2})$?

4. Suppose a surface patch in $\mathbb{R}^3$ is defined by bilinearly interpolating from four vertices. Derive the following formulas for the partial derivatives of $\mathbf{u}$:

$$\frac{\partial \mathbf{u}}{\partial \alpha} = (1 - \beta)(\mathbf{y} - \mathbf{x}) + \beta(\mathbf{z} - \mathbf{w})$$

$$\frac{\partial \mathbf{u}}{\partial \beta} = (1 - \alpha)(\mathbf{w} - \mathbf{x}) + \alpha(\mathbf{z} - \mathbf{y}).$$

In addition, give the formula for the normal vector to the patch at a point $\mathbf{u} = \mathbf{u}(\alpha, \beta)$.

5. This problem is about points in $\mathbb{R}^2$, and their homogeneous representations; and how they act under linear coordinates. Let

$$\mathbf{x} = \langle 0, 0, 2 \rangle \quad \text{and} \quad \mathbf{y} = \langle 4, 8, 4 \rangle$$

be homogeneous representations for the following two vectors in $\mathbb{R}^2$:

$$\mathbf{a} = \langle 0, 0 \rangle \quad \text{and} \quad \mathbf{b} = \langle 1, 2 \rangle.$$

(a) What point $\mathbf{u}$ in $\mathbb{R}^2$ is equal to $\frac{1}{4}\mathbf{a} + \frac{3}{4}\mathbf{b}$?

(b) What point $\mathbf{w}$ in $\mathbb{R}^2$ is represented by (in homogeneous representation) $\frac{1}{4}\mathbf{x} + \frac{3}{4}\mathbf{y}$?

(c) Give values α and β so that $\alpha\mathbf{x} + \beta\mathbf{y}$ is an affine combination giving a homogeneous representation of the point $\mathbf{u}$ calculated in part (a).