Math 173A Discussion 7

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November 13, 2019

1. Consider Newton's method to optimize the function $F(x) = \frac{x^2}{2} + x - \epsilon log(x)$ on $\mathbb{R}^+$, where $1 > \epsilon > 0$ is a given constant.

   (a) Write down explicitly what the Newton update is.

   (b) Suppose $\hat{x}$ is a minimizer of F. Show there exists some sequence $\{\beta_k\}$ such that

   $$x^{(k+1)} - \hat{x} = \beta_k (x^{(k)} - \hat{x})^2$$

2. Let $B \succ 0$. Given a constant matrix $A \in \mathbb{R}^{m \times n}$ and $b \in \mathbb{R}^m$, formulate the dual to the following primal problem:

   $$\begin{aligned} \min_x \quad & x^T B x \\ \text{s.t.} \quad & Ax \leq b \end{aligned}$$

3. Show that strong duality fails to hold for the following problem with domain $D = \{(x, y) : y > 0\}$:

   $$\begin{aligned} \min_{x,y \in D} \quad & e^{-x} \\ \text{s.t.} \quad & \frac{x^2}{y} \leq 0 \end{aligned}$$