Math 173A Discussion 2

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October 9, 2019

1. Fix $n \geq 1$, and let $\phi : \mathbb{R}^n \to \mathbb{R}$ be defined by

$$\phi(x) = v + c^T x + \frac{1}{2} x^T H x$$

where $v \in \mathbb{R}$, $c \in \mathbb{R}^n$, and $H \in \mathbb{R}^{n \times n}$ symmetric.

Compute all derivatives of ϕ.

2. For a fixed $n \geq 1$ and $v \neq 0 \in \mathbb{R}^n$, solve the optimization problem:

$$\begin{aligned} \min_{x \in \mathbb{R}^n} \quad & x^T v \\ \text{s.t.} \quad & ||x||_2 = 1 \end{aligned}$$

Additionally, give the minimizer. Justify each step carefully.