1. Let $f_1, f_2, ..., f_n$ be the convex functions, and $w_1, w_2, ..., w_n$ be nonnegative real numbers. Show that a nonnegative weighted sum of convex functions

$$f = w_1 f_1 + ... + w_n f_n$$

is convex.

2. Show that if f_1 and f_2 are convex functions, then their pointwise maximum f, defined by

$$f(x) = \max\{f_1(x), f_2(x)\}$$

with $\mathbf{dom} f = \mathbf{dom} f_1 \cap \mathbf{dom} f_2$ is also convex.

3. Suppose $h : \mathbb{R} \mapsto \mathbb{R}$ and $g : \mathbb{R}^n \mapsto \mathbb{R}$ are two convex twice differentiable functions. Is the composition function $f = h \circ g$ convex? If so, prove it. If not, give a counter example. If we further assume h is nondecreasing, is the composition function $f = h \circ g$ convex? If not, give a counter example. If so, prove it. If not, give a counter example.

4. Write down the first- and second-order necessary optimality conditions for the function

$$f(x) = x^4(1 + x^2 + \sin x)$$

Hence discuss the optimality of the point $\bar{x} = 0$.

5. Show $f(x) = \sqrt{x}$ is not Lipschitz continuous.

6. Show the logistic loss $f(x) = \log(1 + e^{-x})$, $x \in \mathbb{R}$, is Lipschitz continuous, and find the Lipschitz constant L.