MATH 109, **Exam 2** **May 22, 2017**

Name and PID: _____

Instructions:

- Write your Name and PID.
- You may not use any electronic devices, textbooks, or notes. If you violate these instructions or communicate in any way with any other student during this exam, you will receive a zero on the exam, and the zero will not be dropped.
- **You must justify your work (legibly) to receive credit. In particular, this means that you should state the hypothesis and conclusions of any results that you use.**

Good Luck!

Problems:

1. *[20 points]* Prove or disprove the following statements:
   (a) $\exists y \in \mathbb{R}, \forall x \in \mathbb{R}, xy = 1$.
   (b) $(\exists q \in \mathbb{Z}, n = 2q + 1) \implies (\exists p \in \mathbb{Z}, n^2 = 2p + 1)$.
2. *[25 points]* Let $f : \mathbb{R}^{\geq} \to \mathbb{R}$ and $g : \mathbb{R}^{\geq} \to \mathbb{R}^{\geq}$ be given by $f(x) = x^2$ and $g(x) = x^3 + 1$.
   (a) Find the functions $f \circ g$ and $f \circ f$ *if they can be defined.*
   (b) Determine whether f is injective, surjective, or bijective.
3. *[20 points]* Suppose that $f : X \to Y$ and $g : Y \to Z$ are injections. Prove that $g \circ f : X \to Z$ is also an injection.
4. *[15 points]* Suppose we select any five points inside a square whose sides are of length 1 meter. Use the pigeon-hole principle to show that there exist at least two points within $\frac{\sqrt{2}}{2}$ of each other. **Hint:** Consider dividing the square into smaller squares.
5. *[20 points]* Use the inclusion-exclusion principle to determine how many elements of $\{1, 2, ..., 100\}$ are either multiples of 3 or multiples of 5.