MATH 109, **Exam 1** **April 24, 2017**

Name and PID: _____

Instructions:

- Write your Name and PID.
- You may not use any electronic devices, textbooks, or notes. If you violate these instructions or communicate in any way with any other student during this exam, you will receive a zero on the exam, and the zero will not be dropped.
- You must justify your work (legibly) to receive credit. In particular, this means that you should state the hypothesis and conclusions of any results that you use.

Good Luck!

Problems:

1. *[15 points]* Using a truth table to show that, for all statements P and Q,

   the statement '$P \Rightarrow Q$' is equivalent to the statement '(P or Q) $\Leftrightarrow Q$'.

2. *[15 points]* Prove that for integers a, b, c.

   (a divides b) and (a divides c) $\Rightarrow$ (a divides $b + c$).

3. *[20 points]* Prove the following statements.

   i) For all real numbers x and y, $x^2 + y^2 \geq 2xy$.

   ii) For all real numbers a, b, c, d, $(a^2 + b^2)(c^2 + d^2) \geq (ac + bd)^2$. (You may use the result of the first part here).

4. *[15 points]* Prove by induction that 2 divides $3^n - 1$ for all integers $n \geq 1$.

5. *[15 points]* Given sets A and B, their *symmetric difference* is defined as

   $$A \Delta B = (A - B) \cup (B - A).$$

   Prove that $A \Delta B = (A \cup B) - (A \cap B)$.

6. *[20 points]* Let u_n be the sequence defined by $u_1 = u_2 = u_3 = 1$, and $u_n = u_{n-1} + u_{n-2} + u_{n-3}$, when $n \geq 4$. Prove that for all positive integers n, $u_n < 2^n$.