Midterm 1

② displacement = $\int$ velocity

so $x(t) = \int_0^1 50 + 5te^{-t^2}\,dt$

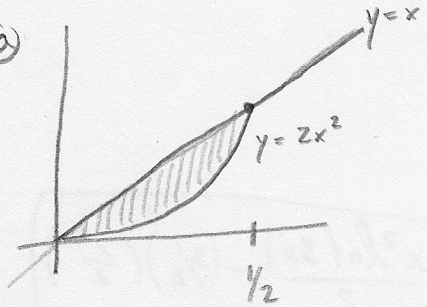
$u = t^2$

$du = 2t\,dt \Rightarrow 5/2\,du = 5t\,dt$

$x(t) = 50t\Big|_0^1 + 5/2\int_0^1 e^{-u}\,du$

$= \boxed{50 + 5/2\left(1 - \frac{1}{e}\right)}$

③ (a)



Intersection when $x = 2x^2$, i.e.

$0 = 2x^2 - x \Rightarrow 0 = (x)(2x-1)$, so when

$x = 0$ & $x = 1/2$.

Area $= \int f_{upper} - f_{lower}$, so

Area $= \int_0^{1/2} x - 2x^2\,dx = \frac{x^2}{2} - \frac{2}{3}x^3\Big|_0^{1/2}$

$= \frac{1}{8} - \frac{2}{3}\left(\frac{1}{8}\right) = \boxed{\frac{1}{24}}$

(b) Volume $= \pi\int$ (radius$_{outer}^2$ − radius$_{inner}^2$), so

Volume $= \pi\int_0^{1/2} x^2 - 4x^4\,dx = \pi\left(\frac{x^3}{3} - 4\frac{x^5}{5}\Big|_0^{1/2}\right)$

$= \pi\left(\frac{1}{24} - \frac{4}{5}\left(\frac{1}{32}\right)\right)$

$= \boxed{\pi/60}$

(4) Average value of f on $[a,b]$ is given by

$$\frac{\int_a^b f(t)\,dt}{b-a}$$

Given $a(t) = 10$, we deduce

$$v(t) = \int_0^t 10\,dx = 10t$$

Therefore, average velocity during first 9 seconds is

$$\int_0^9 \frac{10t\,dt}{9} = \frac{5t^2}{9}\Big|_0^9 = \boxed{45}$$

(5) @ We proceed using integration by parts:

$$u = \ln(3x) \qquad dv = x\,dx$$
$$du = \frac{3}{x}dx \qquad v = \frac{x^2}{2}$$

$$\Rightarrow \int x\ln(3x)\,dx = \frac{x^2\ln(3x)}{2} - \frac{3}{2}\int x\,dx = \boxed{\frac{x^2\ln(3x)}{2} - \left(\frac{3}{2}\right)\left(\frac{x^2}{2}\right)} + C$$

Don't forget your constants folks!

(b) Again, we invoke integration by parts

$$u = e^{2x} \qquad dv = \sin(x)\,dx$$
$$du = 2e^{2x}dx \qquad v = -\cos(x)$$

$$\Rightarrow \int e^{2x}\sin(x)\,dx = -\cos(x)e^{2x} + 2\int e^{2x}\cos(x)\,dx$$

integrating by parts yet again...

$u = e^{2x}$ $\quad dv = \cos(x)\,dx$

$du = 2e^{2x}\,dx$ $\quad v = \sin(x)$

$$\Rightarrow \int e^{2x}\cos(x)\,dx = e^{2x}\sin(x) - 2\int e^{2x}\sin(x)\,dx$$

Plugging this back in to our first equation gives us

$$\int e^{2x}\sin(x)\,dx = -e^{2x}\cos(x) + 2\left[e^{2x}\sin(x) - 2\int e^{2x}\sin(x)\,dx\right]$$

$$= -e^{2x}\cos(x) + 2e^{2x}\sin(x) - 4\int e^{2x}\sin(x)\,dx$$

$$\Rightarrow 5\int e^{2x}\sin(x)\,dx = -e^{2x}\cos(x) + 2e^{2x}\sin(x)$$

$$\Rightarrow \int e^{2x}\sin(x)\,dx = \boxed{\frac{-e^{2x}\cos(x) + 2e^{2x}\sin(x)}{5}} + C$$

⑥ Area $= \frac{1}{2}\int r_{outer}^2 - r_{inner}^2$

$r = 4\cos\theta$

$\Rightarrow r^2 = 4r\cos\theta$

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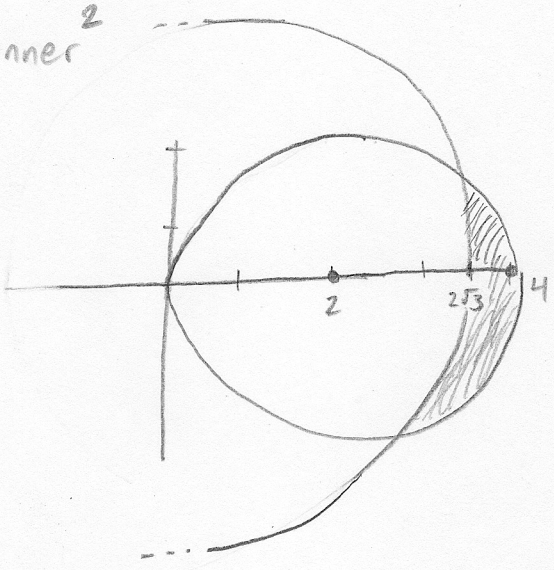
$x^2 + y^2 = 4x$

$\Rightarrow x^2 - 4x + y^2 = 0$

$\Rightarrow (x-2)^2 + y^2 = 4$



Intersect when

$4\cos\theta = 2\sqrt{3}$

$\Rightarrow \cos\theta = \frac{\sqrt{3}}{2}$

$\theta = -\pi/6, \pi/6$

Our integral then is

symmetry! (this follows from $\cos(-\theta) = \cos(\theta)$)

$$\text{Area} = \frac{1}{2}\int_{-\pi/6}^{\pi/6} (4\cos\theta)^2 - (2\sqrt{3})^2 \, d\theta = \frac{1}{2}2\int_{0}^{\pi/6} (4\cos\theta)^2 - (2\sqrt{3})^2 \, d\theta$$

$$= \frac{1}{2}2\int_{0}^{\pi/6} 16\cos^2\theta - 12 \, d\theta$$

Using the nifty double angle formula, we know

$$\cos^2\theta = \frac{1+\cos(2\theta)}{2}$$

Therefore our integral is equal to

$$\frac{1}{2}2\int_{0}^{\pi/6} 8 + 8\cos(2\theta) - 12 \, d\theta = \frac{1}{2}2\left[-4\theta + 4\sin(2\theta)\right]\Big|_{0}^{\pi/6}$$

$$= \frac{1}{2}2\left[-4\pi/6 + 4\sin(\pi/3)\right]$$

$$= \frac{1}{2}2\left[-4\pi/6 + 4(\sqrt{3}/2)\right]$$

$$= \frac{-4\pi/3 + 4\sqrt{3}}{2}$$