3D Floorplan Representations: Corner Links and Partial Order

Fang Qiao*, Ilgweon Kang*, Daniel Kane*, Fung-Yu Young†, Chung-Kuan Cheng*, Ronald Graham*

*University of California, San Diego, La Jolla, CA 92037

†The Chinese University of Hong Kong, Shatin, Hong Kong

Email: *{faqiao, i1kang, dakane, ckcheng, graham}@ucsd.edu, †fyyoung@cse.cuhk.edu.hk

***Abstract*— Data/algorithmic representations of 3D floorplans for integrated circuits is an essential problem in the study of 3D VLSI circuits. Given a fixed number of rectangular blocks and their volume, 3D floorplan representations describe their orientations and positions relative to the origin in a three dimensional space. In our study, we 1). present and analyze a novel 3D floorplan representation we call *corner links*, 2). give new analysis to the partial order representation, and 3). discuss the equivalence of the two representation, provide algorithms for their mutual reducibility, and inspect several key properties.**

Keywords*—*3D Design Methodology; 3D floorplan; floorplan representation

I. Introduction

Floorplaning is the first step in physical design for VLSI circuits. As such, the spatial structuring of non-overlapping blocks, or block-packing, is an important theoretical research topic. In engineering and practice, for flooplan generation and optimization, a 3D representation is crucial to ensure a high quality design. The representation is the foundation for communication between the engineering teams and is the base of the floorplanning algorithms. In theoretical efforts, without efficient and complete methods for floorplan representation, deriving efficient and correct floorplan generation algorithms becomes extremely difficult, as we are limited in our capacity to computationally study VLSI circuit designs.

There has been a growing emphasis on the importance of floorplan representations [1] in the past two decades. Much advancement have been made in 2D floorplan representations, in particular, our new 3D representation described here is inspired by the corner stitching [8] [9] and twin binary trees [14] [13] representations, the latter of which is capable of describing the precise number of mosaic floorplans given fixed number of blocks and their dimensions. Research on 3D floorplan representations is more recent, with increasing interest arising from applications in dynamically reconfigurable FPGA designs and 3D integrated circuits. Existing representations include sequences [12], representation for 3-D slicing floorplan [2], corner block lists [7], bonded slice-line grid [17], T-tree [15] and partial order [16] (examined in this paper in relation to our new method), tree based approach and sequences [11] [10] [4], and topological structures [6]. A comparative study of most of the aforementioned methods and more was done by Fischbach et. al. [3]

In this paper, we introduce a new 3D floorplan representation: **corner links**, and discuss its relation to the **partial order** representation. We will analyze the following four properties (black arrows in Fig 1):

1) Corner links representations can be reduced to partial order representations
2) For non-degenerate floorplans, the corner links representation can be equivalently expressed by a set of **four trees**
3) If a partial order describes relationships between all pairs of blocks in a representation, then the representation corresponds to a valid floorplan
4) The partial order representation yields all cutting planes in a floorplan, in sorted order by their respective dimensions

In addition, we provide algorithms for the following transformations (green arrows in Fig. 1):

1) corner links to partial order
2) partial order to block coordinates in floorplan

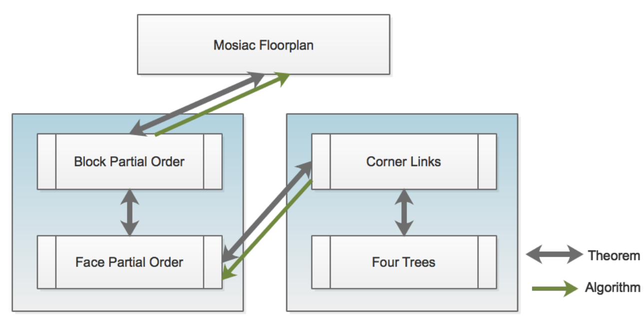


Fig. 1. Overview of article contribution.

II. Floorplan Representations Overview

There are 3 classifications of floorplans:

1) **General** floorplans encompass any floorplan
2) **Slicing** floorplans are floorplans that can be reproduced by recursively subdividing blocks with new boundaries perpendicular to a dimensional axis
3) **Mosaic** floorplans describe non-slicing floorplans without empty spaces

A key property of the mosaic floorplan worth noting is that the topology of mosaic floorplans remains the same after adjusting block sizes by shifting T-junction corners [5], unless the neighboring corners are nondegenerate (Fig. 2.a). This work mainly focuses on mosaic floorplan representations, as currently, there is no previous work for mosaic representations beyond 2D.

Examples of simple floorplan representations include:

- **Absolute coordinate floorplan specifications** - A naïve representation that can be used for visualization purposes is to specify the position and dimensions of each block in a fixed 3D space. However, it's difficult to use this approach to describe and/or analyze interesting properties of various floorplans.
- **Corner-stitching** - A well-known representation for floorplan representations for 2D is corner-stitching, where for opposite corners of every block (e.g. lower-left and upper-right, Fig. 2.b), we track a set of all neighboring blocks. Corner links is inspired by this representation.

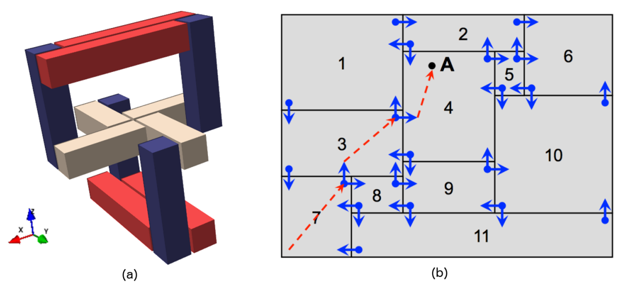


Fig. 2. Floorplan concept illustrations. (a) Nondegenerate neighbors - for the four gray blocks in the center, no block can shift against another due to the relative position of the red and blue outer blocks. (b) Corner-stitching - each blue arrow depicts stitch, which is pointer. Red dotted line shows the searching path for the block containing point A.

III. 3D Floorplan Representations

Now we introduce the 3D representations covered by this paper - corner links and partial order.

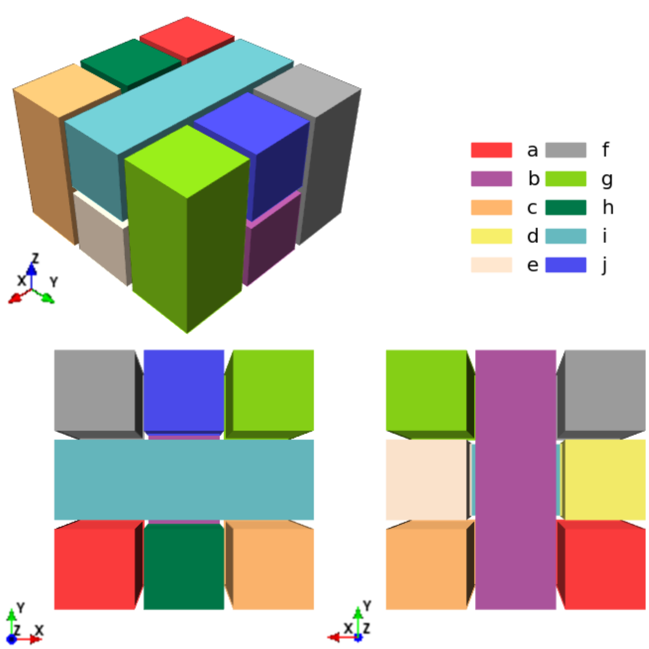


Fig. 3. A simple floorplan example. Top-left: isometric view. Top-right: block labels. Bottom-left: top (z-axis) view. Bottom-right: bottom (z-axis) view.

A. Corner Links

Corner links is our newly formulated 3D floorplan representation. It describes all sets of neighboring corners of blocks in a fixed floorplan. A set of **neighboring corners** is a particular set of all corners in a floorplan that are pair-wise equal by the corner equation. For the example in Fig. 3, there are 36 such sets, e.g. block i's X^-, Y^+, Z^+ (simplified to i^{-+-+}) is linked to the c^{-++} corner, e^{-++} is linked to j^{---}, etc. We write the **corner equation** $e^{-++} = j^{---}$ to indicate pair of neighboring corners.

One interesting case of the corner links representation is for non-degenerate floorplans. In this case, corner links is equivalent to four trees rooted at opposite corners (e.g. the $--+$, $+++$, $+-+$ and $-++$ corners) of the floorplan, similar to the mechanism of twin-binary trees for the 2D case (Fig. 4). See section IV-B for further illustration of the details of the four trees representation in relation to the blocks.

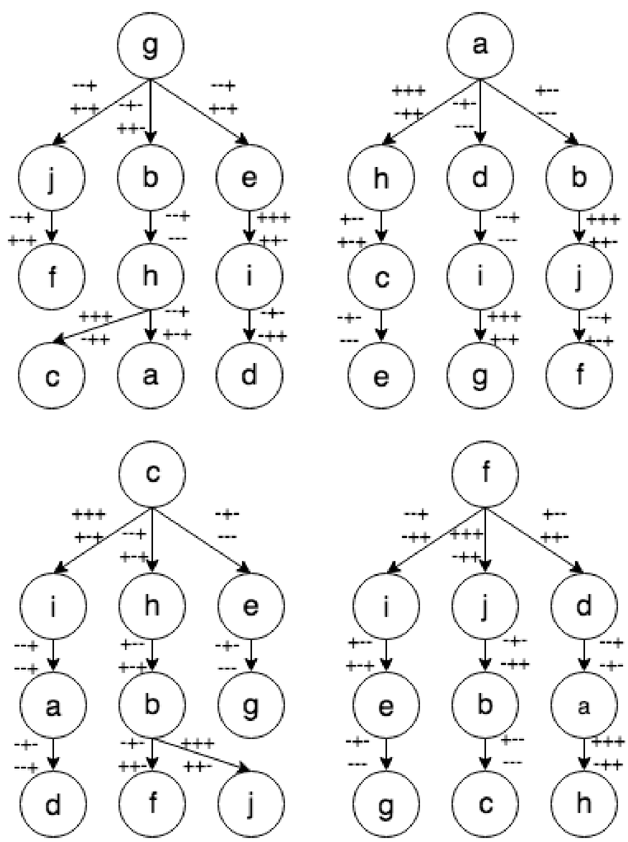


Fig. 4. Four trees representation of the floorplan in Fig. 3.

B. Partial Order

The partial order representation, as described by Yuh et al. [16], describes the topology of a floorplan with three transitive closure graphs for the three dimensions (Fig. 5).

In this paper, we refer to the partial order of the beginning and ending of each block in each dimension as **face partial order**, e.g. the face x-partial order is the transitive closure of the relations given by defining as equal any two touching yz-faces of cubes, and the two yz-faces of the same cube are ordered in the natural way. We refer to the partial order for

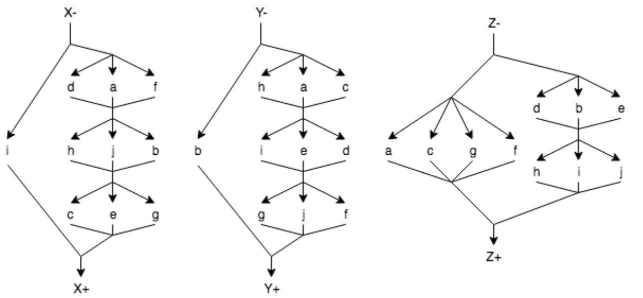


Fig. 5. Block partial order representation of the floorplan in Fig. 3.

individual blocks as partial order/**block partial order**, e.g. for the x-partial order, a cube a is bigger than a cube b iff a's smaller face is at least as large as b's larger face in the face partial order.

IV. 3D Floorplan Representation Properties

A. Corner Links Representation Yields Partial Order

Theorem: The partial order representation of a 3D floorplan can be constructed from the corner links representation, by enumerating all blocks in p^- and p^+ for every **cutting plane** p, a plane characterized by the transitive closure of face equalities coming from faces that touch at a corner, in each of the three dimensions.

For illustration, consider the mosaic 3D floorplan in Fig. 3. For block i's z^- face, b, note that (Fig. 6) b lies in a cutting plane p in dimension z.

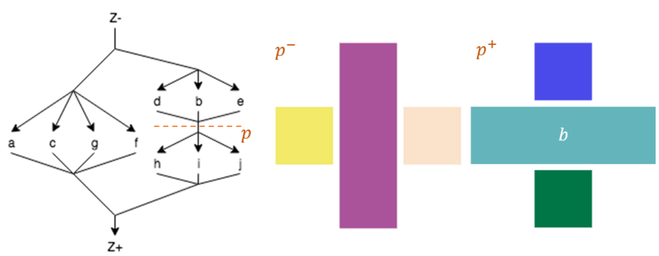


Fig. 6. A cutting plane in the example in Fig. 3. The two blueprints on the right are the footprints above and below the cutting plane p.

Now, if we take the corner link representation equivalence class (dictated by the aforementioned corner equations) of b, we obtain all the blocks that has a face in p. Furthermore, we are able to distinguish the set of blocks in p^- from the set of blocks in p^+. We denote the two sets by E and S, respectively.

Lemma 1: Every corner in a mosaic floorplan, other than those in the eight corners of the floorplan space themselves, must have an odd number of neighboring corners.

At every neighborhood of corners, the largest number of corners we can have is eight (each corner takes up an eighth of the neighborhood). The non-corner neighbors consist of edges that take up two corners, and faces that takes up four corners. So each non-corner neighbor can only take up an even number of corners, leaving an even number of corner in the neighborhood. Therefore, each corner must have an odd number of corner neighbors.

Lemma 2: The footprints of E and S on the cutting plane must match.

If the footprint of E and S do not match, then take the symmetric difference of the two footprints with respect to the corner equations. Take an arbitrary bottom-left corner c of this symmetric difference. Because c is the result of a symmetric difference, it would mean that it has a even number of corner neighbors. But c must have an odd number of corner neighbors due to Lemma 1. (Fig. 7).

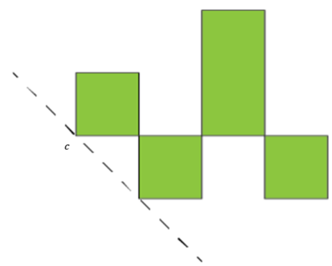


Fig. 7. Illustration of symmetric difference for footprints across a cutting plane that supposedly do not match. 1). The symmetric difference is a non-empty set. 2). The layout of the symmetric difference has some lower left corner, c. and since it represents the difference of two sets, it means it has an even number of neighboring corners (each pair of corners above and below the cutting plane eliminates each other in the difference).

Therefore, the symmetric difference of the footprints of S and E must be empty. The footprints must match.

As such, by repeatedly taking the equivalence classes of all corners in a dimension that do not have a cutting plane accounted for, we can enumerate all cutting planes and generate the face partial orders of a floorplan from the corner links representation.

B. The Four Trees Representation is Equivalent to Corner Links for Non-Degenerate Floorplans

Theorem: For non-degenerate mosaic floorplans, four trees representation is sufficient for unique representation.

For illustration, consider an arbitrary floorplan. We specify our four trees representations to be rooted at the $--+$, $+++$, $+-+$ and $-++$ corners or the floorplan, and outgoing connections (edges) from each block (vertex) can be sourced at the $--+$, $+++$, $+-+$ and $-++$ corners of the block. We arbitrarily call these red corners, and others black corners (Fig. 8.a).

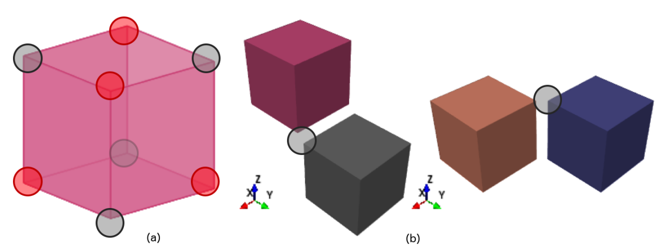


Fig. 8. Illustration of blocks corners for the purpose of the four trees representation: (a). red corners can have outgoing edges in the four trees representation to any neighboring corners in the floorplan, black corners do not have outgoing edges; (b). for any two neighboring black corners in a mosaic floorplan, there must be a red corner in the same neighborhood.

Consider any arbitrary corner intersection within the floorplan. There must be at least one corner that is a block's red corner, for the black corners are positioned opposite each other at the intersection, and would therefore guarantee the existence of a red corner (Fig. 8.b).

Hence, we can say without loss of generality that every mosaic floorplan is described by a corresponding four trees representation. Conversely, since the corner links/four trees representation uses corner equations, each four trees representation describes a single mosaic floorplan.

C. Characterization of Valid Floorplans in the Partial Order Representation

We say a floorplan defined in a representation is valid if no two blocks overlap each other in space.

Theorem: Given any two blocks in a mosaic floorplan, they are related to each other in at least one of the three block partial orders.

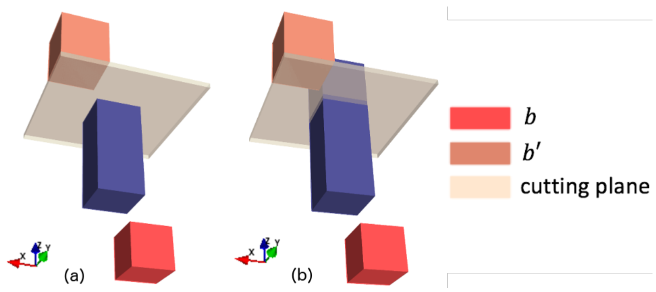


Fig. 9. Paths between blocks in partial order: (a). blue block have a face on the cutting plane - starting from b^{+++}, we can trace the side of the blue block until it reaches the z-dimension cutting plane at b'^{---}, so there is a path in the z partial order from b to b'; (b). blue block goes through the cutting plane - starting from b^{+++}, we can trace the side of the blue block until it reaches the z-dimension cutting plane, and switch to another partial order and continue tracing until we reach b'^{---}. Whichever partial order we switched to after crossing the cutting plane (either y or x in this case) contains the path from b to b'.

For illustration, consider an arbitrary mosaic 3D floor-plan. Fix some block b and its $+++$ corner, c. There are 3 cutting planes, one in each partial order, at that corner.

Consider some other corner c' of some block b' that comes after block b in one of the three partial orders.

Lemma 3: If two blocks meet at a corner, then the corresponding faces at that corner are related in all three of the face partial orders.

Lemma 4: If two rectangles meet at a corner in 2D, then the corresponding faces at that corner are related in both face partial order of the 2D plane. Note this is the 2D case of Lemma 3.

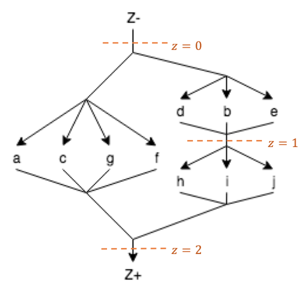
Starting from corner c of block b, we can trace $+++$ corners until we reach some block that intersects one of the three cutting planes at c'. Note that by Lemma 3, each of the three $+++$ planes of this block are at least the corresponding $+++$ planes of b in the partial orders of the respective dimensions. There are then two cases:

a. The block we reach has a face on the cutting plane. b' has a path to b in that dimension. (Fig. 9.a)
b. The block we reached crosses the cutting plane. Then, by Lemma 4, the block is connected to b in the partial order of at least one of the remaining dimensions other than that of the plane we reached. So b has a path to b in that dimension. (Fig. 9.b)

In either case, a path exists from b to b' in at least one partial order.

D. Face Partial Order Representation Consists of All Cutting Planes In Sorted Order

The face partial order of each dimension provides information on the relative order the blocks start and ends in that dimension, and thus the cutting planes. So for each dimension, we can iterate through the cutting planes, incrementing a “layer counter” for each cutting plane, and obtain the starting and ending coordinate of each block (Fig. 10).



blocks	z^-	z^+
a, c, g, f	0	2
d, b, e	0	1
h, i, j	1	2

Fig. 10. Obtaining z dimension coordinates of all blocks from the z partial order of the example in Fig. 3.

V. 3D Floorplan Representation Algorithms

A. Corner Links to Face Partial Order Representation

Algorithm 1 Corner links to face partial order representation conversion algorithm

Input: $CornerLinks$ ▷ Corner links representation, consists of a list of sets of equivalent block corners
Output: $POrders$ ▷ 3 lists of cutting planes ordered to each dimension

For each $cornerLink$ in $CornerLinks$, we obtain three cutting planes
From the corners of each block in $CornerLinks$, we obtain the ordering of the cutting planes in each dimension
$POrder$ is then the transitive closure of the previous two relations

B. Face Partial Order to Absolute Coordinate Representation

Algorithm 2 Face partial order representation to absolute coordinates of a mosaic floorplan conversion algorithm

Input: $POrder$ ▷ 3 lists of blocks ordered to each dimension
Output: $Coords$ ▷ For every block, we give the coordinates of the $---$ and $+++$ corners.

For each dimension $pOrder$ in $POrder$, topologically sort the $pOrder$ and assign a layer number to each cutting plane
Assign the beginning and ending coordinates of each block to their layer numbers from the previous step

VI. CONCLUSION

In this paper, we have presented our new 3D floorplan representation, corner links, and gave analysis of its properties along with that of the existing partial order representation. Specifically, we discussed 1). the relationship between corner links and partial order; 2). the four tree representation as a special case in corner links, 3). properties of admissible floorplans in the partial order representation; and 4). transformation of partial order representation to absolute coordinates of blocks in a mosaic floorplan.

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