MATH 100A MIDTERM #1 100 pts Ron Evans October 26, 2011

Directions: Justify all answers (including true-false).
Notation: Here G is always a multiplicative group with identity 1, and S_n denotes the group of $n!$ permutations of n symbols.
Points: #1, 3, 4, 8: Twelve pts each; #5, 6, 7: Fifteen pts each; #2: Seven pts.

(1) Let G be the direct product of the two groups $(Z/9Z)^*$ and $(Z/7Z)^*$.
(A) How many elements are in G? (B) True or False: G is a cyclic group.

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(2) Let $\sigma = (1235)(426) \in S_6$. What is the order of the permutation σ ?

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(3) Let G be a cyclic group generated by an element g of order 120.
List all the elements of order 20 in G.

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(4) Students A, B, C, D, E are waiting in line in that order. Every minute two random students switch places in line. Is it possible that after 37 switches, the students end up in line in the reverse order E, D, C, B, A ?
Hint: Look at what the permutation (15)(24) does to the ordering of the students.

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(5) Let a and b be elements of an abelian group G. If a and b each have finite order, explain in detail why their product ab must also have finite order.

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(6) Let h be an element of H, where H is a subgroup of G. Prove that hH = H.

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(7) Let g be an element in G. If g has order m and if $g^n = 1$, prove that m divides n. *Note:* Lagrange's theorem is *not* needed for the proof. Nevertheless, you are welcome to use Lagrange's theorem if you insist, provided you prove it first.

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(8) Show that S_5 has twenty different elements of order 3. *Hint:* 5*4*3 = 60.
EXTRA CREDIT (if you have extra time): How many elements of order 3 are in S_6 ?