Math 171A Homework Assignment #6

Due Date: 11pm, Monday, February 28, 2022

Caution: Show major computational steps and proofs in submitted solutions! Mathematical reasons should be given to justify the answers. Any solutions obtained with usage of any kind of software are not acceptable!

1. (10 points) Consider the iLP

\[
\begin{aligned}
\text{min} & \quad c^T x \\
\text{s.t.} & \quad Ax \geq b,
\end{aligned}
\]

where \( A, b, c \) are given as

\[
A = \begin{bmatrix}
1 & 1 & 1 \\
0 & 2 & -3 \\
0 & 3 & -2 \\
1 & -1 & -1
\end{bmatrix},
\quad b = \begin{bmatrix}
1 \\
1 \\
0 \\
1
\end{bmatrix},
\quad c = A^T \begin{bmatrix}
1 \\
1 \\
0 \\
1
\end{bmatrix}.
\]

Observe the expression of \( c = A^T \lambda \) in the above and use optimality conditions to guess a minimizer for this iLP, and then verify that it is a minimizer.

2. (10 points) Consider the iLP:

\[
\begin{aligned}
\text{min} & \quad c^T x \\
\text{s.t.} & \quad Ax \geq b
\end{aligned}
\]

where \( A, b, c \) are given as

\[
A = \begin{bmatrix}
1 & 1 & 1 \\
1 & -1 & 2 \\
1 & -2 & 1 \\
1 & 2 & 2
\end{bmatrix},
\quad b = \begin{bmatrix}
3 \\
-1 \\
-1 \\
5
\end{bmatrix},
\quad c = \begin{bmatrix}
2 \\
3
\end{bmatrix}.
\]

For the point \( u = (1, 4/3, 2/3) \), determine whether or not there exists \( \lambda \) such that

\[
\lambda = \begin{bmatrix}
\lambda_1 \\
\lambda_2 \\
\lambda_3 \\
\lambda_4
\end{bmatrix} \geq 0, \quad c = A^T \lambda, \quad \lambda^T (Au - b) = 0.
\]

3. (10 point) Consider the mixed constrained LP

\[
\begin{aligned}
\text{min} & \quad 3x_1 + 5x_2 + 7x_3 + 5x_4 \\
\text{s.t.} & \quad x_1 + x_2 + x_3 + x_4 = 2, \\
& \quad x_1 + x_2 \geq 1, \\
& \quad x_2 + x_3 \geq 1, \\
& \quad x_3 + x_4 \geq 1, \\
& \quad x_1 + x_4 \geq 1.
\end{aligned}
\]

Determine whether or not \( u = (0, 1, 0, 1) \) is a minimizer of the above LP.
4. (10 points) Consider the standard LP:

\[
\begin{align*}
\min & \quad c^T x \\
\text{s.t.} & \quad Ax = b, \quad x \geq 0,
\end{align*}
\]

where \(A, b, c\) are given as

\[
A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & -3 & 2 \end{bmatrix}, \quad b = \begin{bmatrix} 1 \\ -2 \end{bmatrix}, \quad c = \begin{bmatrix} -1 \\ 0 \\ -2 \\ 0 \end{bmatrix}.
\]

Compute all vertices of the feasible set. For each vertex, determine whether it is degenerate or nondegenerate. Determine which vertex is a minimizer.

5. (10 points) Consider the following linear program

\[
\begin{align*}
\min & \quad 2x_1 + 2x_2 + x_3 + x_4 \\
\text{s.t.} & \quad 2x_1 + x_2 + 2x_3 + x_4 = 3, \\
& \quad x_1 + 2x_2 + x_3 + 2x_4 = 3, \\
& \quad x_1 \geq 0, x_2 \geq 0, x_3 \geq 0, x_4 \geq 0.
\end{align*}
\]

Use the Simplex method to solve the above LP with the starting vertex \(x^{(0)} = (1,1,0,0)\).