Math 171B Homework Assignment #3

Due date: 11pm, April 25, 2022

The first 5 questions are required to be submitted in Gradescope.

1. (10 points) Let $f(x)$ be the function in $x \in \mathbb{R}^3$ such that

$$f(x) = x_1^4 + x_2^4 + x_3^4 - 2x_1^2x_2^2 - 3x_1^2x_3^2 - 5x_2^2x_3^2.$$ 

Find the gradient and Hessian of the composition function $h(x) = f(Ax)$ for the matrix

$$A = \begin{bmatrix} 3 & -2 & -1 \\ 0 & 2 & -1 \\ 0 & 0 & 1 \end{bmatrix}.$$ 

2. (10 points) Determine whether or not the following function is convex

$$f(x) := x_1^4 + x_2^4 + 4x_1^2x_2.$$ 

3. (10 points) Determine the range of values for $a$ so that

$$f(x) := x_1^2 + 2x_2^2 + 3x_3^2 + 2x_1x_2 + 2x_1x_3 + ax_2x_3$$ 

is a convex function in $\mathbb{R}^3$.

4. (10 points) Using the bisection method to compute a sequence of intervals $\{[a_k, b_k]\}$ approaching a zero point of $f(x) := \cos \left( \frac{\pi}{2} x \right)$ on the interval $[-1.3, 3.7]$. Compute $[a_k, b_k]$ for $k = 1, 2, 3$. What is the limit of $s \left\{ \frac{1}{2}(a_k + b_k) \right\}_{k=1}^\infty$?

5. (10 points) Use Newton’s method to solve the equation

$$x - \frac{3}{x} = 0$$ 

with the starting point $x_0 = 1$. Write down the iteration formula, and compute $x_1, x_2, x_3$. 

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The following questions are optional. Do NOT submit them in Gradescope.

6. Use Secant’s method to solve the equation
\[ x - \frac{3}{x} = 0 \]
with the starting point \( x_0 = 1, x_1 = 2 \). Write down the iteration formula, and compute \( x_2, x_3, x_4 \). Express them in rational numbers.

7. Apply Newton’s method to solve equation \( x^3 - a = 0(a > 0) \), with initial guess \( x_0 > 0 \). If the Newton’s sequence converges, what is its convergence order? Justify your answer.

8. Let \( f : \mathbb{R}^n \to \mathbb{R} \) be a twice continuously differentiable function. Define the composite function \( \varphi(t) = f(u + td) \), with given vectors \( u, d \in \mathbb{R}^n \). Express \( \varphi'(t) \) and \( \varphi''(t) \) in terms of the gradient and Hessian of \( f \) and the vectors \( u, d \).

9. Let \( f(x) \) be a three times differentiable function and \( f(x^*) = f'(x^*) = 0, f''(x^*) \neq 0 \). Suppose \( \{x_k\}_{k=0}^\infty \) is a sequence generated by Newton’s method and converges to \( x^* \). Show that it has convergence order 1.

10. Apply Newton’s method to solve equation \( x^* = 0 \) with initial guess \( x_0 = 1 \). Does the Newton’s sequence converge to a zero point? If it does, what is the limit and what is the convergence order? If it does not, give your reasons.