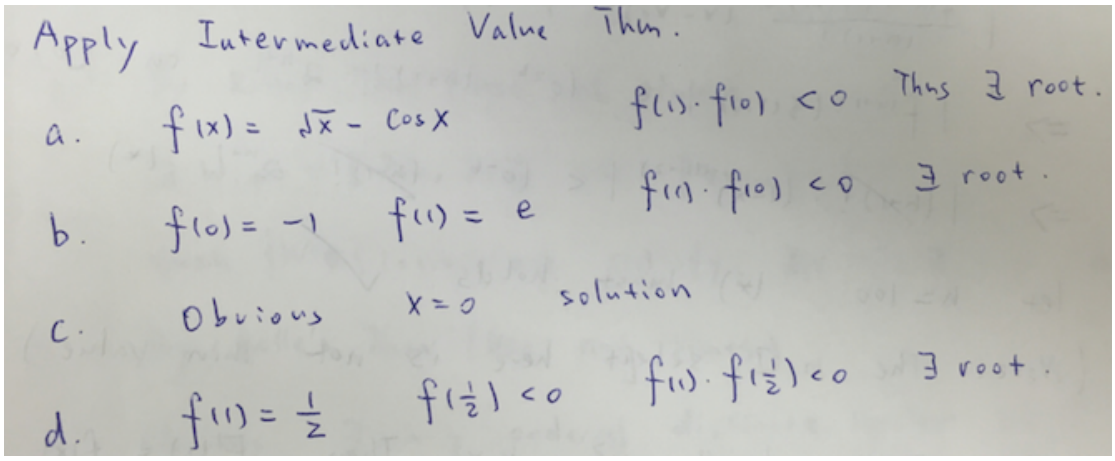
MATH 170B SP2016

HW1 Solution

Question2：



Question4：

c. Find intervals that contain a solution to the equation $x^3 - 2x^2 - 4x + 3 = 0$.

SOLUTION: Let $f(x) = x^3 - 2x^2 - 4x + 3$. The critical points of f occur when

$$0 = f'(x) = 3x^2 - 4x - 4 = (3x + 2)(x - 2);$$

that is, when $x = -\frac{2}{3}$ and $x = 2$. Relative maximum and minimum values of f can occur only at these values. There are at most three solutions to $f(x) = 0$, because $f(x)$ is a polynomial of degree three. Since $f(-2) = -5$ and $f\left(-\frac{2}{3}\right) \approx 4.48$; $f(0) = 3$ and $f(1) = -2$; and $f(2) = -5$ and $f(4) = 19$; solutions lie in the intervals $[-2, -2/3]$, $[0, 1]$, and $[2, 4]$.

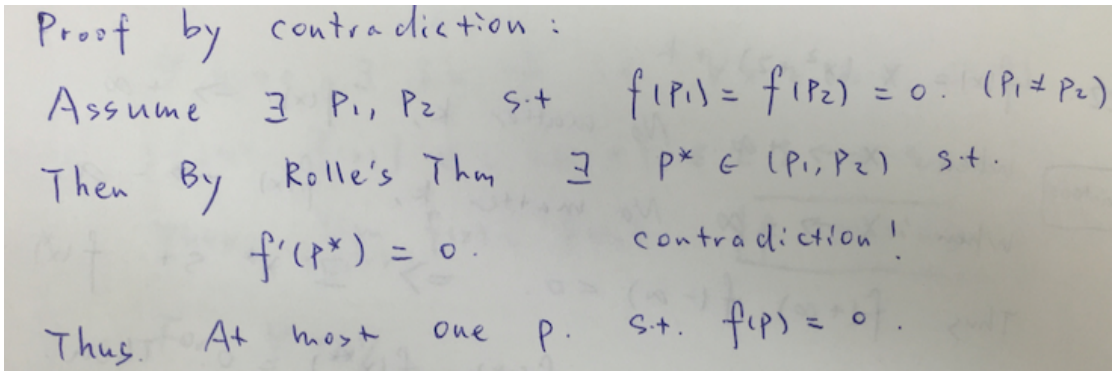
Question6：

a. Find $\max_{0 \le x \le 1} |f(x)|$ when $f(x) = (2 - e^x + 2x)/3$.

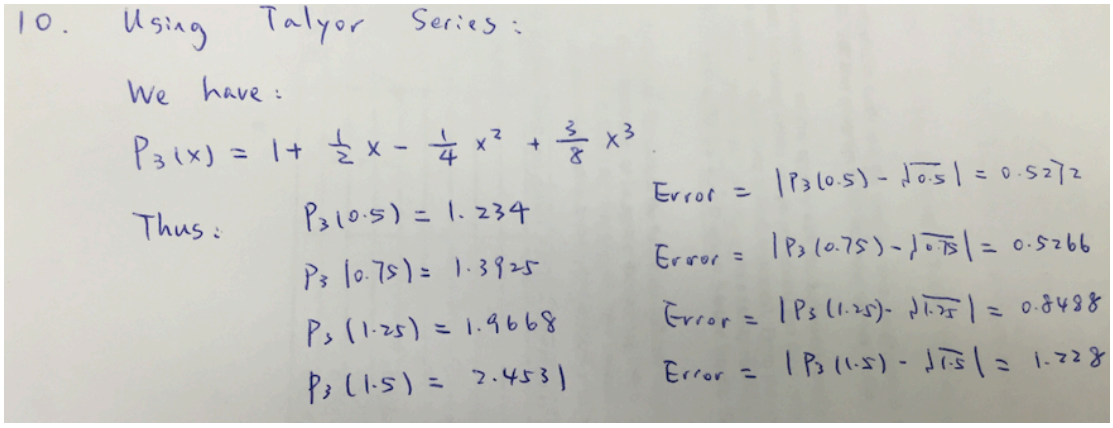
SOLUTION: First note that $f'(x) = (-e^x + 2)/3$, so the only critical point of f occurs at $x = \ln 2$, which lies in the interval $[0, 1]$. The maximum for $|f(x)|$ must consequently be

$$\max\{|f(0)|, |f(\ln 2)|, |f(1)|\} = \max\{1/3, (2\ln 2)/3, (4 - e)/3\} = (2\ln 2)/3.$$

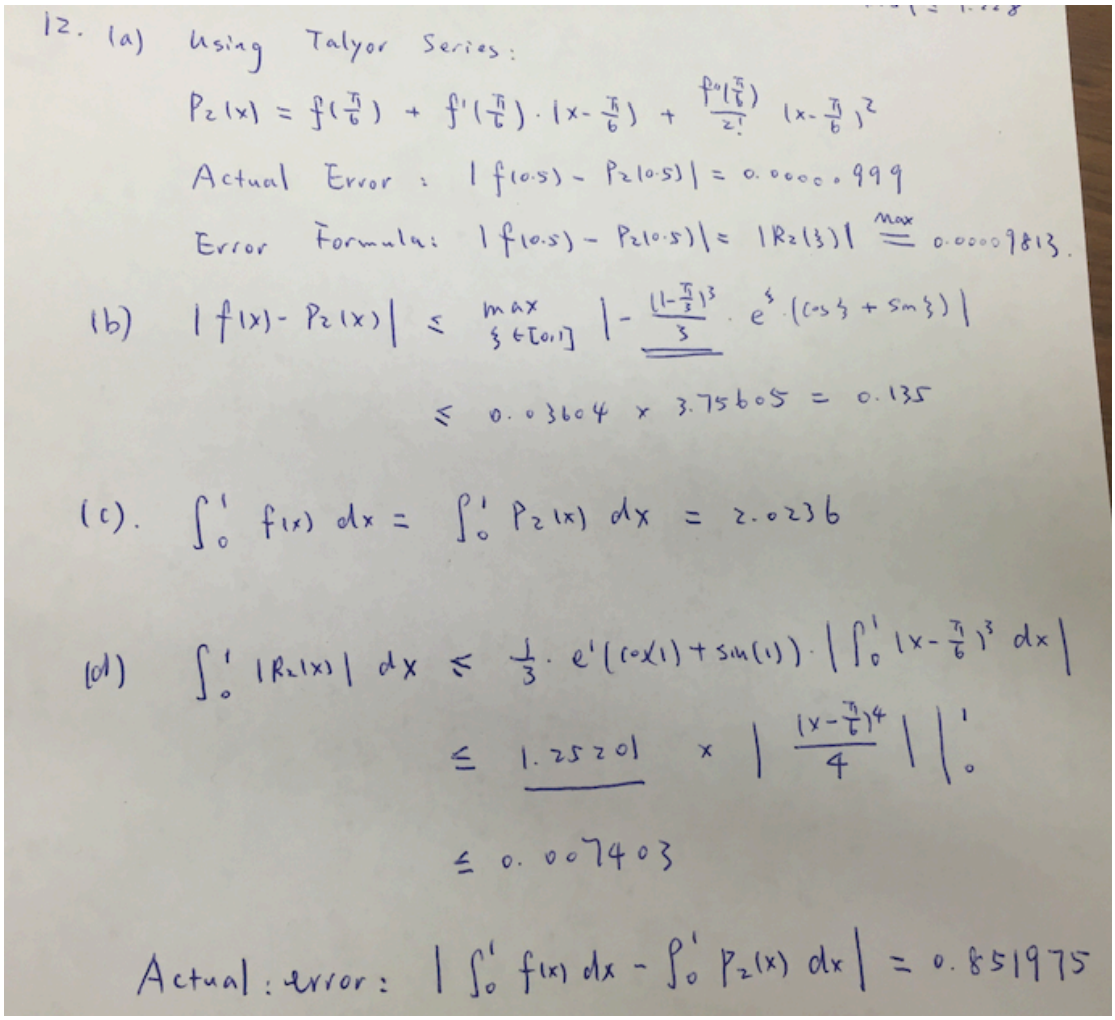
Question8:



Question10:



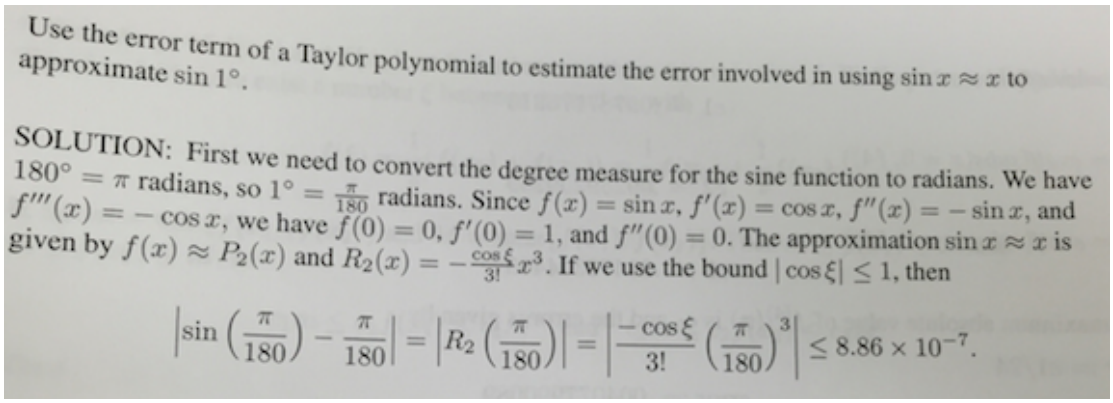
Question12:



Question 14:

Refer solution Question 12.

Question16:

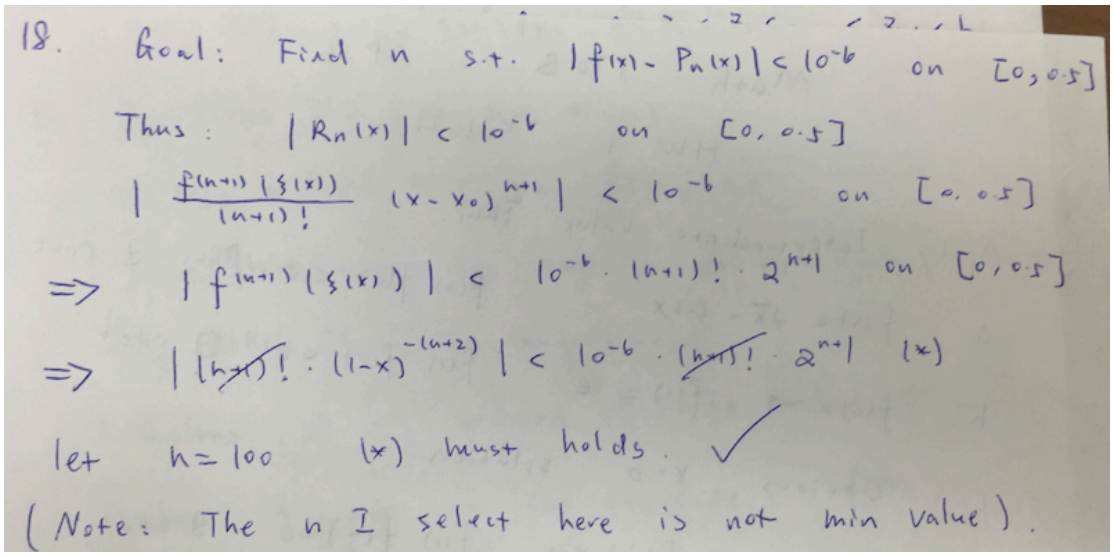


Use the error term of a Taylor polynomial to estimate the error involved in using $\sin x \approx x$ to approximate $\sin 1°$.

SOLUTION: First we need to convert the degree measure for the sine function to radians. We have $180° = \pi$ radians, so $1° = \frac{\pi}{180}$ radians. Since $f(x) = \sin x$, $f'(x) = \cos x$, $f''(x) = -\sin x$, and $f'''(x) = -\cos x$, we have $f(0) = 0$, $f'(0) = 1$, and $f''(0) = 0$. The approximation $\sin x \approx x$ is given by $f(x) \approx P_2(x)$ and $R_2(x) = -\frac{\cos \xi}{3!}x^3$. If we use the bound $|\cos \xi| \leq 1$, then

$$\left|\sin\left(\frac{\pi}{180}\right) - \frac{\pi}{180}\right| = \left|R_2\left(\frac{\pi}{180}\right)\right| = \left|\frac{-\cos\xi}{3!}\left(\frac{\pi}{180}\right)^3\right| \leq 8.86 \times 10^{-7}.$$

Question18:



18. Goal: Find n s.t. $|f(x) - P_n(x)| < 10^{-6}$ on $[0, 0.5]$

Thus: $|R_n(x)| < 10^{-6}$ on $[0, 0.5]$

$$\left|\frac{f^{(n+1)}(\xi(x))}{(n+1)!}(x - x_0)^{n+1}\right| < 10^{-6} \quad \text{on } [0, 0.5]$$

$$\Rightarrow \quad |f^{(n+1)}(\xi(x))| < 10^{-6} \cdot (n+1)! \cdot 2^{n+1} \quad \text{on } [0, 0.5]$$

$$\Rightarrow \quad |(n+1)! \cdot (1-x)^{-(n+2)}| < 10^{-6} \cdot (n+1)! \cdot 2^{n+1} \quad (*)$$

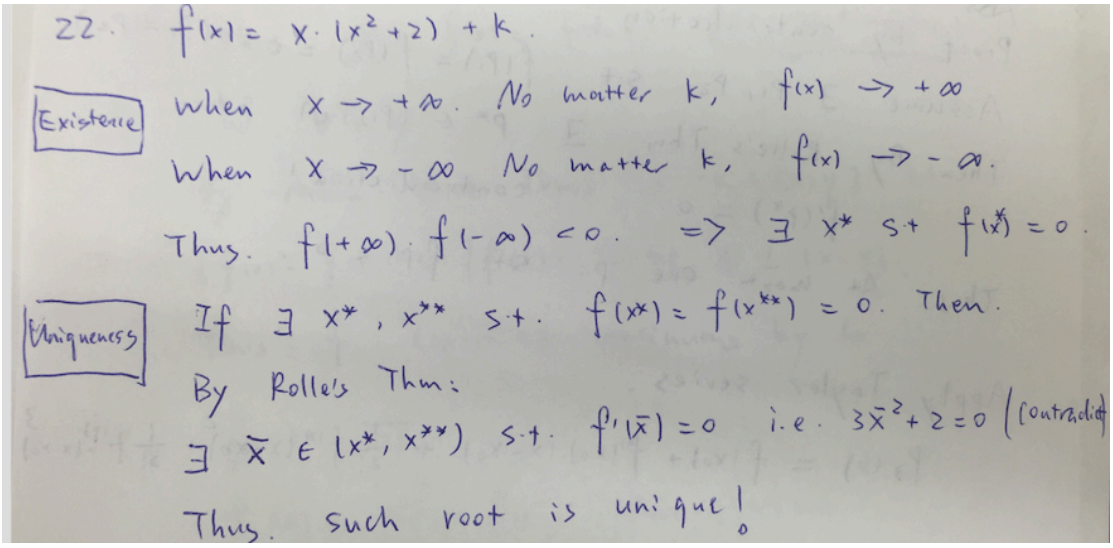
let $n = 100$ $(*)$ must holds. ✓

(Note: The n I select here is not min value).

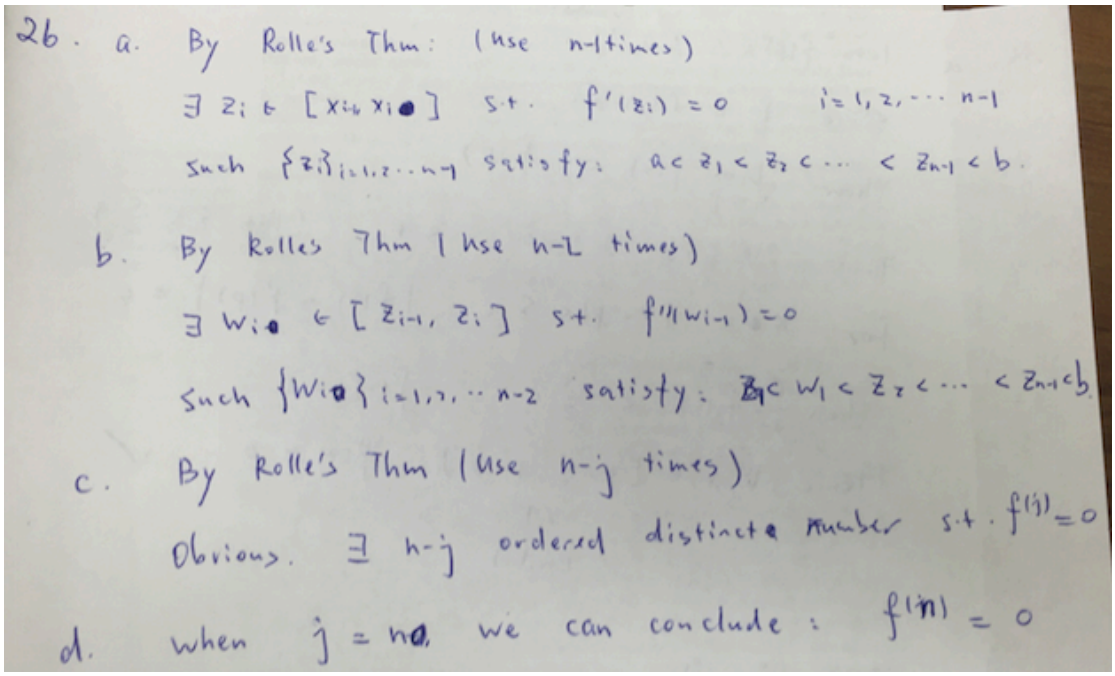
Question20:

Apply Taylor series.

Question22:



Question 26:



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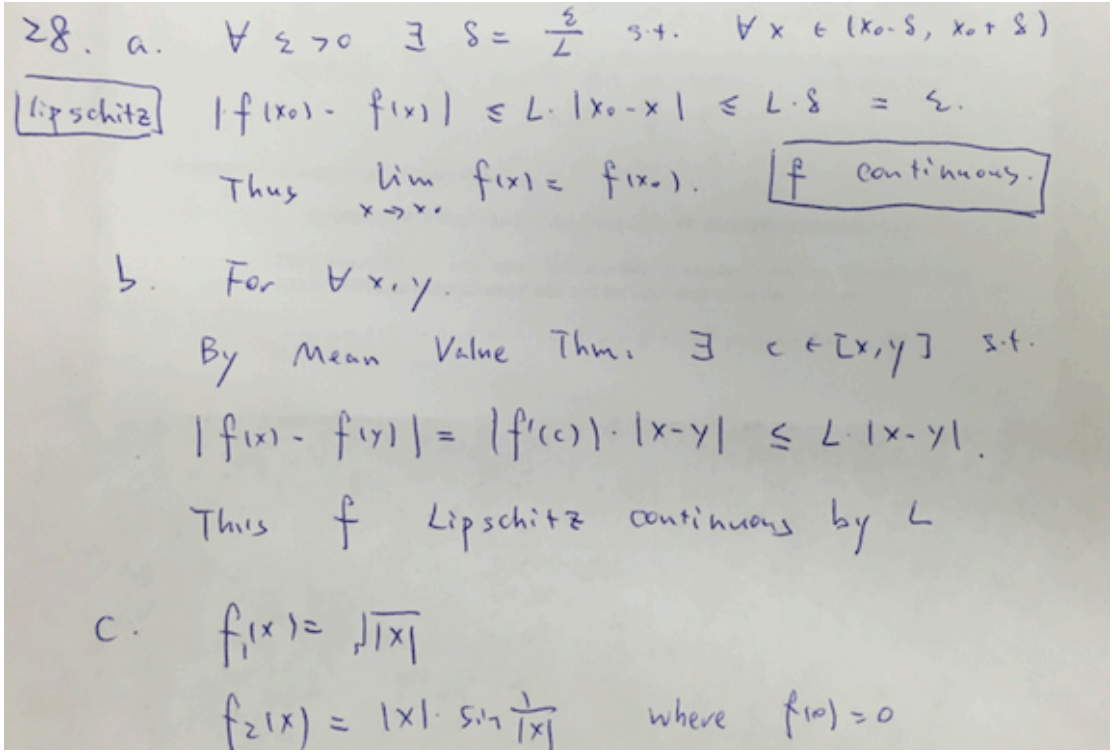
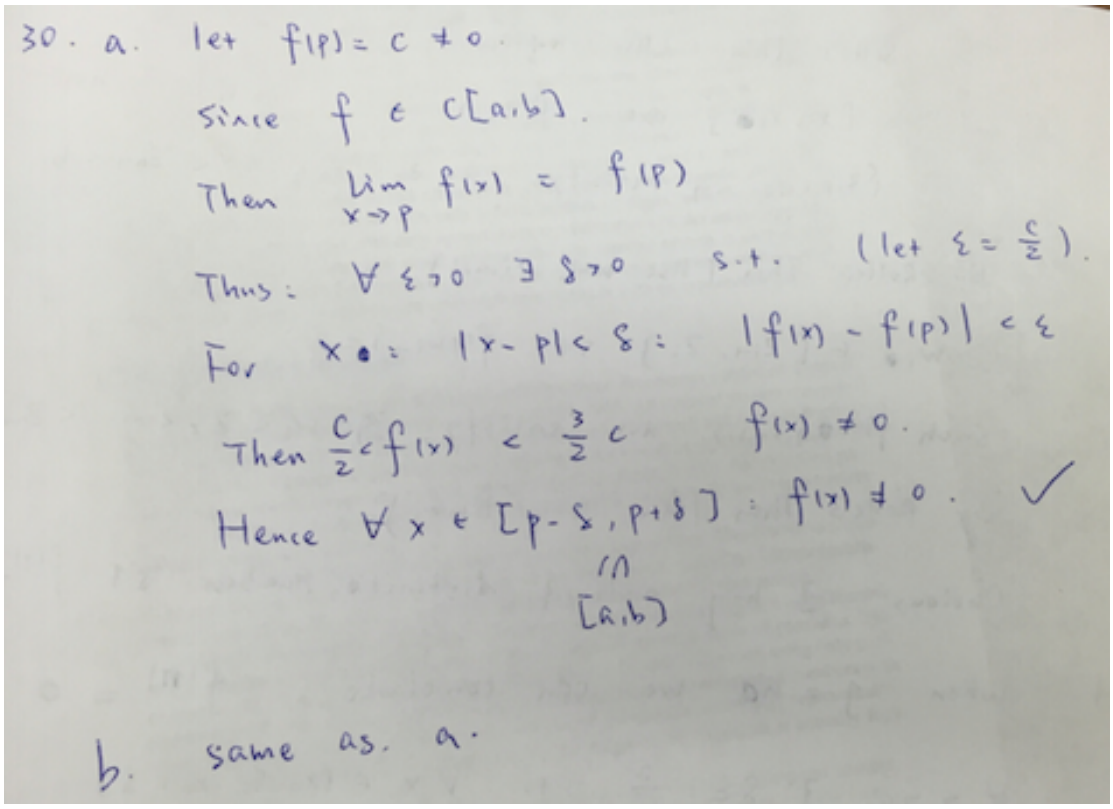
Question30: