Practice Problems Final

Problem 1. Evaluate
$$5(7+3)-2[(6-3)-4^2]+1.$$

Solution. We use PEMDAS here. First we evaluate the first bracket
$$(7+3)=10.$$
So we have
$$5\times 10-2[(6-3)-4^2]+1.$$
We then evaluate the second bracket
$$(6-3)-4^2=3-4^2=3-16=-13.$$
So we have
$$\begin{aligned}&5\times 10-2\times(-13)+1\\&=50-2\times(-13)+1\\&=50-(-26)+1\\&=50+26+1\\&=76+1=77.\end{aligned}$$

Problem 2. Simplify and write your answer with positive exponents:

$$(p^{-4}q^3)^8.$$

Solution. We use the property of exponents to get

$$(p^{-4}q^3)^8 = (p^{-4})^8(q^3)^8 = p^{-32}q^{24} = \frac{q^{24}}{p^{32}}.$$

Problem 3. Convert -8.05×10^{-12} into standard notation.

Solution.

$$-8.05 \times 10^{-12} = -0.00000000000805.$$

Problem 4. Simplify

$$\left(\frac{7}{8}\right)^{-\frac{1}{4}} \times \left(\frac{7}{8}\right)^{\frac{1}{2}} \times \left(\frac{7}{8}\right)^{\frac{3}{4}}.$$

Solution. By the property of exponents, we have

$$\left(\frac{7}{8}\right)^{-\frac{1}{4}} \times \left(\frac{7}{8}\right)^{\frac{1}{2}} \times \left(\frac{7}{8}\right)^{\frac{3}{4}} = \left(\frac{7}{8}\right)^{-\frac{1}{4}+\frac{1}{2}+\frac{3}{4}}.$$

We have

$$-\frac{1}{4} + \frac{1}{2} + \frac{3}{4} = \frac{1}{4} + \frac{3}{4} = 1.$$

So

$$\left(\frac{7}{8}\right)^{-\frac{1}{4}} \times \left(\frac{7}{8}\right)^{\frac{1}{2}} \times \left(\frac{7}{8}\right)^{\frac{3}{4}} = \left(\frac{7}{8}\right)^{1} = \frac{7}{8}.$$

Problem 5. What is the degree of the polynomial $q^7 - 2q + 3q^3 - 9q$? What is the coefficient of q^6 in this polynomial?

Solution. This is a polynomial of degree 7. There is no q^6 term here, so the coefficient of q^6 is 0.

Problem 6. Subtract

$$\frac{3x}{2(x-1)} - \frac{4}{(x-1)(x+2)}.$$

Solution. The denominators here are $2(x-1)$ and $(x-1)(x+2)$, and so the common denominator is $2(x-1)(x+2)$. Therefore we get

$$\frac{3x}{2(x-1)} = \frac{3x}{2(x-1)} \times \frac{x+2}{x+2} = \frac{3x(x+2)}{2(x-1)(x+2)} = \frac{3x^2+6x}{2(x-1)(x+2)}$$

and

$$\frac{4}{(x-1)(x+2)} = \frac{4}{(x-1)(x+2)} \times \frac{2}{2} = \frac{8}{2(x-1)(x+2)}.$$

So we have

$$\frac{3x}{2(x-1)} - \frac{4}{(x-1)(x+2)} = \frac{3x^2+6x}{2(x-1)(x+2)} - \frac{8}{2(x-1)(x+2)} = \frac{3x^2+6x-8}{2(x-1)(x+2)}.$$

Problem 7. Find the distance between the points $(7, 4)$ and $(3, 1)$. Also, find the mid-point of the line joining these two points.

Solution. The distance between two points (x_1, y_1) and (x_2, y_2) is given by

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

So, the distance between $(7, 4)$ and $(3, 1)$ is

$$\sqrt{(7-3)^2 + (4-1)^2} = \sqrt{4^2 + 3^2} = \sqrt{16+9} = \sqrt{25} = 5.$$

The mid-point of the line joining two points (x_1, y_1) and (x_2, y_2) is given by

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right).$$

So, the mid-point of the line joining $(7, 4)$ and $(3, 1)$ is

$$\left(\frac{7+3}{2}, \frac{4+1}{2}\right) = \left(5, \frac{5}{2}\right).$$

Problem 8. The difference in the age of two people is 20 years. If 5 years ago, the elder one of the two was 5 times as old as the younger one, then find their present ages.

Solution. This was done in lecture. Check the notes for Friday (12/8).

Problem 9. The perimeter of a tablet of graph paper is 48 *in*. The length is 6 *in* more than the width . Find the area of the graph paper.

Solution. Let us denote the length by L and the width by W. Then we know the following two facts:

- The perimeter is 48, i.e.
$$2L + 2W = 48.$$
Dividing by 2, we can simplify this to get
$$L + W = 24.$$

- The length is 6 more than the width, i.e.
$$L = W + 6.$$

Substituting $W + 6$ for L in the first equation, we get
$$(W + 6) + W = 24,$$
i.e.
$$2W + 6 = 24.$$
Solving this gives us
$$W = 9.$$
So
$$L = W + 6 = 9 + 6 = 15.$$
Therefore, the area of the graph paper is
$$L.W = 15 \times 6 = 90 \text{ in}^2.$$

Problem 10. Simplify:

$$\frac{1+5i}{-3i}$$

Solution. We multiply the numerator and denominator by the conjugate of $-3i$, i.e. $3i$.

$$\frac{1+5i}{-3i} = \frac{1+5i}{-3i} \times \frac{3i}{3i} = \frac{(1+5i)(3i)}{(-3i)(3i)}.$$

Using distribution, we get

$$\frac{(1+5i)(3i)}{(-3i)(3i)} = \frac{3i+15i^2}{-9i^2}.$$

Since $i^2 = 1$, we get

$$\frac{(1+5i)(3i)}{(-3i)(3i)} = \frac{3i-15}{9} = -\frac{15}{9} + \frac{3}{9}i = -\frac{5}{3} + \frac{1}{3}i.$$

Problem 11. The area of a rectangular plot is 528 m^2. The length of the plot is one more than twice its breadth. Find the length and breadth of the plot.

Solution. Let us denote the length by L and the breadth by B. Then we know the following two facts:

- The area is 528, i.e.

$$L.B = 528.$$

- The length is 1 more than twice the breadth, i.e.

$$L = 2B + 1.$$

Substituting $2B + 1$ for L in the first equation, we get

$$(2B + 1).B = 528,$$

i.e.

$$2B^2 + B - 528 = 0.$$

This is a quadratic equation that can be factored as $2B^2 + B - 528 = (2B + 33)(B - 16)$. So we have

$$(2B + 33)(B - 16) = 0.$$

Solving this gives us

$$B = -\frac{33}{2} \quad \text{or} \quad B = 16.$$

Since the breadth is a positive number, we conclude that $B = 16$. Also

$$L = 2W + 1 = 2 \times 16 + 1 = 33.$$

Therefore, the length of the plot is 16 m and the breadth of the plot is 33 m.

Problem 12. Solve for x:

$$\sqrt{x} + 2 = x.$$

Solution. We take the 2 to the right hand side to get

$$\sqrt{x} = x - 2.$$

Squaring both sides, we get

$$(\sqrt{x})^2 = (x - 2)^2$$

which gives

$$x = x^2 - 4x + 4.$$

Subtracting x from both sides, we get

$$x^2 - 5x + 4 = 0.$$

This can be factored as $x^2 - 5x + 4 = (x - 1)(x - 4)$. So

$$(x - 1)(x - 4) = 0.$$

So we get $x = 1$ or $x = 4$.
We have to check that these are actually solutions. Let us first check $x = 1$. We have

$$\sqrt{1} + 2 = 3,$$

which is not equal to 1. So $x = 1$ is not a solution.
Now let us check $x = 4$. We have

$$\sqrt{4} + 2 = 2 + 2 = 4.$$

Therefore, $x = 4$ is the only solution to this equation.

Problem 13. Solve for x:

$$|x-1| \geq 2.$$

Solution. $|x-1| \geq 2$ implies that either

$$x-1 \geq 2$$

or

$$x-1 \leq -2.$$

The first inequality gives

$$x \geq 3.$$

The second inequality gives

$$x \leq -1.$$

So we have $x \geq 3$ or $x \leq -1$, i.e.

$$x \in (-\infty, -1] \cup [3, \infty).$$

Problem 14. Solve for r:

$$\frac{|3+r|}{7} \leq 5.$$

Solution. Multiplying by 7 on both sides, we get

$$|3+r| \leq 35.$$

This implies that

$$-35 \leq 3+r \leq 35$$

Subtracting by 3 on all sides, we get

$$-38 \leq r \leq 32,$$

i.e.

$$r \in [-38, 32].$$

Problem 15. Solve for a:

$$5a^2 - 5a = 35.$$

Solution. We can first divide both sides by 5 to get

$$a^2 - a = 7.$$

Taking 7 to the left hand side, we get

$$a^2 - a - 7 = 0.$$

We can check that this cannot be factored, and so we use the quadratic formula. We have

$$a = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-7)}}{2} = \frac{1 \pm \sqrt{1+28}}{2} = \frac{1 \pm \sqrt{29}}{2}.$$

Problem 16. Solve for x:

$$\frac{x}{3} + \frac{3}{x} = \frac{17}{4}.$$

Solution. The denominators involved are 3, 4 and x. So the common denominator is $12x$. Multiplying both sides of the equation by $12x$, we get

$$12x\left(\frac{x}{3} + \frac{3}{x}\right) = 12x \times \frac{17}{4}.$$

Distributing and canceling terms, we get

$$4x^2 + 36 = 51x.$$

This gives us a quadratic equation

$$4x^2 - 51x + 36 = 0.$$

We can factorize this as

$$4x^2 - 51x + 36 = (4x - 3)(x - 12).$$

So we have

$$(4x - 3)(x - 12) = 0.$$

Therefore, we get two solutions, i.e. $x = \frac{3}{4}$ and $x = 12$.

Problem 17. Evaluate:

$$\frac{7-i}{2+10i}.$$

Solution. We have to multiply the numerator and denominator by the conjugate of $2+10i$, i.e. $2-10i$.

$$\frac{7-i}{2+10i} = \frac{7-i}{2+10i} \times \frac{2-10i}{2-10i} = \frac{(7-i)(2-10i)}{(2+10i)(2-10i)}.$$

We use FOIL on the numerator and the $(a-b)(a+b) = a^2 - b^2$ identity on the denominator to get

$$\frac{7-i}{2+10i} = \frac{(7-i)(2-10i)}{(2+10i)(2-10i)} = \frac{14-70i-2i+10i^2}{4-100i^2}.$$

Since $i^2 = -1$, we get

$$\frac{7-i}{2+10i} = \frac{14-70i-2i+10i^2}{4-100i^2} = \frac{14-72i-10}{4+100} = \frac{4-72i}{104}.$$

Canceling a factor of 4 from the numerator and denominator, we get

$$\frac{7-i}{2+10i} = \frac{1-18i}{26} = \frac{1}{26} - \frac{9}{13}i.$$

Problem 18. Solve for x:

$$9 - 2(x - 5) = x + 20.$$

Solution. Distributing $2(x - 5)$, we get

$$9 - (2x - 10) = x + 20.$$

Removing the bracket, we get

$$9 - 2x + 10 = x + 20,$$

i.e.

$$19 - 2x = x + 20.$$

Adding $2x$ to both sides gives us

$$19 = 3x + 20.$$

Subtracting 20 from both sides, we get

$$-1 = 3x,$$

which gives us

$$x = -\frac{1}{3}.$$

Problem 19. Suppose that L denotes the line passing through the points $(-9, -3)$ and $(11, 4)$. Find the slope of a line perpendicular to L.

Solution. This was done in lecture. Check the notes for Friday (12/8).

Problem 20. Solve for b:

$$\frac{1}{b^2 - 7b + 10} + \frac{1}{b-2} = \frac{2}{b^2 - 7b + 10}.$$

Solution. We first factorize $b^2 - 7b + 10$. We want to find two integers p, q such that $p + q = -7$ and $p.q = 10$. An easy check gives us that $p = -2$ and $q = -5$. So we have

$$b^2 - 7b + 10 = (b-2)(b-5).$$

Therefore, we rewrite our original equation as

$$\frac{1}{(b-2)(b-5)} + \frac{1}{b-2} = \frac{2}{(b-2)(b-5)}.$$

The denominators involved are $(b-2)(b-5)$ and $b-2$, and hence the common denominator is $(b-2)(b-5)$. Multiplying both sides of the equation with $(b-2)(b-5)$, we get

$$(b-2)(b-5)\left(\frac{1}{(b-2)(b-5)} + \frac{1}{b-2}\right) = (b-2)(b-5)\left(\frac{2}{(b-2)(b-5)}\right).$$

Distributing and canceling terms, we get

$$1 + b - 5 = 2.$$

This gives us

$$b = 6.$$