Homework 4

October 25, 2022

Due by 11:59 pm, Oct 24

Problem 1. Use the method of undetermined coefficients and the superposition principle to solve the following ODE:

\[ y'' + y = \sin t - 3e^{2t} \]

Problem 2. Find the general solution to the following ODEs:

i) \[ y'' - 6y' + 9y = t - 3e^{3t} \]
ii) \[ y'' + y = \tan^2 t \]

Problem 3. The method of variation of parameters can also be used to solve non-homogeneous linear ODEs that needn’t have constant coefficients. The method is pretty much the same. In this question, we’ll work out an example. Consider the ODE:

\[ ty'' + (5t - 1)y' - 5y = t^2e^{-5t} \]

i) Check that \( y_1 = 5t - 1 \) and \( y_2 = e^{-5t} \) are solutions to the corresponding homogeneous equation, i.e.

\[ ty'' + (5t - 1)y' - 5y = 0 \]

ii) Put \( y_p = v_1y_1 + v_2y_2 \) and plug it in to find \( v_1 \) and \( v_2 \). Keep in mind that we don’t want \( v_1'' \) and \( v_2'' \) to show up, so we impose the condition \( v_1'y_1 + v_2'y_2 = 0 \).
iii) Write the general solution as

\[ y(t) = y_p(t) + c_1y_1(t) + c_2y_2(t) \]

Problem 4. Find two linearly independent solutions to the following ODEs:

i) \( t^2y''(t) + 7ty'(t) - 7y(t) = 0 \)
ii) \( y''(t) - \frac{1}{t}y'(t) + \frac{5}{t^2}y(t) = 0 \)