Problem 1. Consider a general linear equation:

\[ \frac{dy}{dx} + P(x)y = Q(x) \]

i) Write it in the form \( M(x, y)dx + N(x, y)dy \) by taking \( Q(x) \) to the LHS and then multiplying the ODE by \( dx \).

ii) Use the above form to check when it will be exact.

iii) In general, find an integrating factor \( \mu \) to make this ODE exact.

Problem 2. Solve the following IVP:

\[ (3x^2 y + 2xy + y^3)dx + (x^2 + y^2)dy = 0, \quad y(0) = 1 \]

Problem 3. Solve the following IVP:

\[ 6y'' - 5y' + y = 0, \quad y(0) = 1, \quad y'(0) = 1 \]

Problem 4. Solve the following IVP:

\[ y'' + 4y' + 4y = 0, \quad y(0) = 1, \quad y'(0) = 3 \]

Problem 5. Find the general solution to the following ODEs:

i) \( y'' + y = 0 \)

ii) \( 2y'' + 2y' + y = 0 \)
Problem 6. Consider the ODE:

\[ y''' - y'' - y' + y = 0 \]

i) Write the characteristic equation associated to this ODE and find the solution(s) to that equation.

ii) Analogous to the 2nd order situation, try to find 3 distinct non-zero solutions to the ODE and check that all three are solutions.

iii) Guess what the general solution should be.