P1

\[ \frac{dy}{dx} = (1 + y^2)\tan(x) \]
\[ \int \frac{dy}{1 + y^2} = \int \tan(x) dx \]
\[ \tan^{-1}(y) = \ln|\sec(x)| + C \]

P2

\[ \frac{d(x/y)}{dx} = 1/y \]
\[ \frac{d(1+ln(y))}{dy} = 1/y \]

Hence exact. Now anti-derivative \( \int (x/y)dy = xln(y) + k_1(x) \int 1+ln(y)dx = x + xln(y) + k_2(y) \)

Hence \( x + xln(y) = C \) is the solution, for some constant \( C \)

P3

\[ (x^4 - x + y)dx - xdy = 0 \]
\[ x(dy/dx) - (x^4 - x + y) = 0 \]
\[ dy/dx - y/x = x^3 - 1 \]

integrating factor \( e^{\int -1/x dx} = 1/x \) Hence solution

\[ y = \frac{\int (x^3-1)(1/x)dx + C}{1/x} \]
\[ = \frac{x^3}{3} - ln(x) + Cx \]
\[ = x^4/3 - xln(x) + Cx \]

P4

\[ y_c = C_1e^{-2t} + C_2e^{-t} \]

Undetermined coefficient, guess \( y_p = Ae^{-3t} \). Plug into diff eqn we see that \( 9A - 9A + 2A = 2 \) and therefore \( A = 1 \). Hence general solution \( y = y_p + y_c = e^{-3t} + C_1e^{-2t} + C_2e^{-t} \)
P5

\[ y'' + 4y = 2\tan(2t) - e^t \]
\[ y_c = C_1\cos(2t) + C_2\sin(2t) \]

Variation of parameters to get \( y_{p1} \) for the \( 2\tan(2t) \). Use formula at P189 of the textbook.

\[ v_1 = \int -(2\tan(2t))\sin(2t)dt \quad v_2 = \int (2\tan(2t))\cos(2t)dt \]

Then use undetermined coeff to get \( y_p = \frac{-1}{5}e^t \) for the \( e^t \). Now superposition principle

\[ y = v_1\cos(2t) + v_2\sin(2t) - \frac{1}{5}e^t + C_1\cos(2t) + C_2\sin(2t) \]

P6

characteristic eqn \( r(r-1) + 7r + 5 = 0 \)
\[ r^2 + 6r + 5 = 0 \]
Hence \( y = C_1t^{-5} + C_2t^{-1} \)
\[ y' = -5C_1t^{-6} - C_2t^{-2}. \]
Hence to solve \( C_1, C_2 \), we have \(-1 = C_1 + C_2\) \( 13 = -5C_1 - C_2 \)
\[ C_1 = -3, C_2 = 2 \]

P7

\[ y'' + \frac{1-2}{t}y' + \frac{t-1}{t}y = 0 \]
\[ y'' + (\frac{1}{t} - 2)y' + (1 - \frac{1}{t})y = 0 \]
\[ y'' + p(t)y + q(t) = 0 \]

Reduction of order as we are already GIVEN \( y_1 = e^t \). Simply use the equation on textbook P197

\[ y_2 = y_1 \int \frac{e^{-\int p(t)dt}}{y_1^2}dt \]
\[ = e^t \int \frac{e^{-\left(ln(t) - 2t\right)}}{e^{2t}}dt \]
\[ = e^t \int t^{-1}dt \]
\[ = e^t \ln(t) \]