## A MATH FONT SAMPLER

MICHAEL SHARPE

This document displays on each of the subsequent pages a single path of mathematical text such as one might find in a mix of textbooks. The text fonts are mostly not free, and the math fonts are a mix of free and non-free. The point was to show what other options are available, sometimes at either modest monetary cost, or zero monetary cost and substantial effort. The text fonts that appear here are:

- Minion Pro (Adobe).
- Utopia (Adobe, available in TEX Live).
- Libertine (Public, available in TEX Live).
- Bergamo (Fontsite) - a clone of Adobe Bembo.
- Savoy (Fontsite) - a clone of Adobe Sabon.
- Barbedor (Fontsite) - a clone of URW Barbedor.
- Garamond Premier Pro (Adobe).
- University OldStyle (Fontsite) - a clone of ITC Berkeley OldStyle.
- Caslon Pro (Adobe).
- Jenson Recut (Fontsite) - a clone of Monotype Centaur.
- Hoefler Text (Apple).
- Arno Pro (Adobe).
- Goudy Old Style (Softkey).
- Jenson Pro (Adobe).
- Warnock Pro (Adobe).

The math fonts are:

- Lucida New Math (not free.)
- txfonts (Public-part of TEX Live
- Euler (Public-part of TEX Live.)
- mathpazo (Public-part of TEX Live.)
- Mathtime Pro 2 Lite (pctex, free Lite version.) + Mathematica bold
- MnSymbol with MinionPro (MnSymbol is public-part of TEXLive.)

In addition to mtpro2 Lite with its Times letters, some of the fonts are paired with a variant of mtpro2 Lite which replaces the letters with those from another text font. The virtual math fonts produced by this method have names like $\mathbf{z 5 s b m t}$, in which the initial character z , in the Berry fontname scheme, stands for a singular case not following the normal naming rules, 5sb is the Berry three-character code for Fontsite Savoy, and mt
signifies Mathtime. These frankenfonts were produced by TeXFontUtility, free from
http://math.ucsd.edu/~msharpe/TeXFontUtility.dmg
which provides an interface to the fontinst scripts and the parameters that must be passed to it. (It also provides interfaces to other fontinst scenarios and to otfinst.)

The main drawback to mtpro2 lite is, in my opinion, the lack of a good quality bold Greek alphabet. As you will see in the examples, when the bold Greek letters have to be synthesized, they do not usually provide a good match to other characters, and their shapes are sometimes awkward. It may be worth buying the full version just to get around this issue. The ionPro\}Aninversionformula:Let$g:\mathbb{R}^{+}\rightarrow\mathbb{R}$beboundedandrightcontinuous,andlet$\varphi(\alpha):=\int_{0}^{\infty}e^{-\alphat}g(t)dt$denoteitsLaplacetransform.Then,forevery$t>0$,undefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefined

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\begin{equation*}
g(t)=\lim _{\varepsilon \rightarrow 0} \lim _{\lambda \rightarrow \infty} \varepsilon^{-1} \sum_{\lambda t<k \leq(\lambda+\varepsilon) t} \frac{(-1)^{k}}{k!} \lambda^{k} \varphi^{(k)}(\lambda) \tag{1}
\end{equation*}
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Solutions of systems of ODEs: Let $\mathbf{v}(\mathbf{x}, \boldsymbol{\alpha})$ denote a parametrized vector field $(\mathbf{x} \in U, \boldsymbol{\alpha} \in A)$ where $U$ is a domain in $\mathbb{R}^{n}$ and the parameter space $A$ is a domain in $\mathbb{R}^{m}$. We assume that $\mathbf{v}$ is $C^{k}$-differentiable as a function of $(\mathbf{x}, \boldsymbol{\alpha})$, where $k \geq 2$. Consider a system of differential equations in $U$ :

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\begin{equation*}
\dot{\mathbf{x}}=\mathbf{v}(\mathbf{x}, \boldsymbol{\alpha}), \quad \mathbf{x} \in U \tag{2}
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Fix an initial point $\mathbf{p}_{0}$ in the interior of $U$, and assume $\mathbf{v}\left(\mathbf{p}_{0}, \boldsymbol{\alpha}_{0}\right) \neq \mathbf{0}$. Then, for sufficiently small $t,\left|\mathbf{p}-\mathbf{p}_{0}\right|$ and $\left|\boldsymbol{\alpha}-\boldsymbol{\alpha}_{0}\right|$, the system (2) has a unique solution $\mathbf{x}_{\boldsymbol{\alpha}}(t)$ satisfying the initial condition $\mathbf{x}_{\boldsymbol{\alpha}}(0)=\mathbf{p}$, and that solution depends differentiably (of class $C^{k}$ ) on $t, \mathbf{p}$ and $\boldsymbol{\alpha}$.

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\begin{equation*}
\Gamma(z) \sim e^{-z} z^{z-1 / 2} \sqrt{2 \pi}\left[1+\frac{1}{12 z}+\frac{1}{288 z^{2}}-\frac{139}{51840 z^{3}}+\cdots\right], \quad z \rightarrow \infty \text { in }|\arg z|<\pi \tag{3}
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Bézier curves: Given $z_{1}, z_{2}, z_{3}, z_{4}$ in $\mathbb{C}$, define the Bézier curve with control points $z_{1}, z_{2}, z_{3}, z_{4}$ by

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```
\usepackage{libertine}
\usepackage[T1]{fontenc}
\usepackage[subscriptcorrection,slantedGreek]{zfxlmt}
\usepackage[leqno]{amsmath} %use eq no on left
```

An inversion formula: Let $g: \mathbb{R}^{+} \rightarrow \mathbb{R}$ be bounded and right continuous, and let $\varphi(\alpha):=\int_{0}^{\infty} e^{-\alpha t} g(t) d t$ denote its Laplace transform. Then, for every $t>0$,

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\usepackage[lining] {bergamo}
\usepackage{eulervm}% with lining figures for math roman
\renewcommand*{\rmdefault}{5bbj}% set osf after math loaded
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\usepackage[lining] {bergamo}
\usepackage[expert,vargreek] {lucbmath}
\def\DeclareLucidaFontShape#1#2#3#4#5#6{%
    \DeclareFontShape{#1}{#2}{#3}{#4}{<->s*[0.8]#5}{#6}}% reduce to 80%
\renewcommand*{\rmdefault}{5bbj}% set osf after math loaded
```

An inversion formula: Let $g: \mathbb{R}^{+} \rightarrow \mathbb{R}$ be bounded and right continuous, and let $\varphi(\alpha):=\int_{0}^{\infty} e^{-\alpha t} g(t) d t$ denote its Laplace transform. Then, for every $t>0$,

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```
\usepackage[lining]{savoy}
\usepackage[lite,subscriptcorrection,slantedGreek]{z5sbmt}% with lining figures for math roman
\renewcommand*{\rmdefault}{5sbj}% set osf after math loaded
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\usepackage[lining] {savoy}
\usepackage[expert,vargreek] {lucbmath}
\def\DeclareLucidaFontShape#1#2#3#4#5#6{%
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An inversion formula: Let $g: \mathbb{R}^{+} \rightarrow \mathbb{R}$ be bounded and right continuous, and let $\varphi(\alpha):=\int_{0}^{\infty} e^{-\alpha t} g(t) d t$ denote its Laplace transform. Then, for every $t>0$,

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\usepackage[lining]{universityoldstyle}
\usepackage[expert,vargreek] {lucbmath}
\def\DeclareLucidaFontShape#1#2#3#4#5#6{%
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\usepackage[lining] {adobecaslon}
\usepackage[expert,vargreek] {lucbmath}
\def\DeclareLucidaFontShape#1#2#3#4#5#6{%
    \DeclareFontShape{#1}{#2}{#3}{#4}{<->s*[0.88]#5}{#6}}% reduce to 88%
\renewcommand*{\rmdefault}{pacj}% set osf after math loaded
```

An inversion formula: Let $g: \mathbb{R}^{+} \rightarrow \mathbb{R}$ be bounded and right continuous, and let $\varphi(\alpha):=\int_{0}^{\infty} e^{-\alpha t} g(t) d t$ denote its Laplace transform. Then, for every $t>0$,

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\usepackage[lining] {hoeflertext}
\usepackage[expert,vargreek] {lucbmath}
\def\DeclareLucidaFontShape#1#2#3#4#5#6{%
    \DeclareFontShape{#1}{#2}{#3}{#4}{<->s*[0.83]#5}{#6}}% reduce to 83%
\renewcommand*{\rmdefault}{ehtj}% set osf after math loaded
```

An inversion formula: Let $g: \mathbb{R}^{+} \rightarrow \mathbb{R}$ be bounded and right continuous, and let $\varphi(\alpha):=\int_{0}^{\infty} e^{-\alpha t} g(t) d t$ denote its Laplace transform. Then, for every $t>0$,

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\usepackage[lining] {arno}
\usepackage[T1]{fontenc}
\usepackage[expert,vargreek] {lucbmath}
\def\DeclareLucidaFontShape#1#2#3#4#5#6{%
    \DeclareFontShape{#1}{#2}{#3}{#4}{<->s*[0.83]#5}{#6}}% reduce to 83%
\renewcommand*{\rmdefault}{pa0j}% set osf after math loaded
```

An inversion formula: Let $g: \mathbb{R}^{+} \rightarrow \mathbb{R}$ be bounded and right continuous, and let $\varphi(\alpha):=\int_{0}^{\infty} e^{-\alpha t} g(t) d t$ denote its Laplace transform. Then, for every $t>0$,

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\usepackage[lining]{adobejenson}
\usepackage[expert,vargreek] {lucbmath}
\def\DeclareLucidaFontShape#1#2#3#4#5#6{%
    \DeclareFontShape{#1}{#2}{#3}{#4}{<->s*[0.8]#5}{#6}}% reduce to 80%
\renewcommand*{\rmdefault}{pajj}% set osf after math loaded
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\end{aligned}
$$

Residue theorem: Let $f$ be analytic in the region $G$ except for the isolated singularities $a_{1}, a_{2}, \ldots, a_{m}$. If $\gamma$ is a closed rectifiable curve in $G$ which does not pass through any of the points $a_{k}$ and if $\gamma \approx 0$ in $G$, then

$$
\frac{1}{2 \pi i} \int_{\gamma} f=\sum_{k=1}^{m} n\left(\gamma ; a_{k}\right) \operatorname{Res}\left(f ; a_{k}\right)
$$

Maximum modulus principle: Let $G$ be a bounded open set in $\mathbb{C}$ and suppose that $f$ is a continuous function on $\bar{G}$ which is analytic in $G$. Then

$$
\max \{|f(z)|: z \in \bar{G}\}=\max \{|f(z)|: z \in \partial G\}
$$

Jacobi's identity: Define the theta function 9 by

$$
\vartheta(t)=\sum_{n=-\infty}^{\infty} \exp \left(-\pi n^{2} t\right), \quad t>0
$$

Then

$$
\vartheta(t)=t^{-1 / 2} \vartheta(1 / t)
$$

