Nonlinear Dynamics and Control of Mechanical Systems

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Invited Session Proposal

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The goal of this session is to survey some of the modern trends in nonlinear dynamics and control theory. The unifying theme of this session is the use of intrinsic dynamical properties of mechanical systems in the design of control strategies. In particular, geometric approaches to dynamics, optimization and control are presented. The following topics will be addressed: Hamiltonian systems on matrix spaces and their network structure, energy shaping of mechanical and discrete systems and its applications to digital control, formation control, trajectory design, and motion camouflage.

We now describe in detail the talks in our session.

Ali Belabbas and Roger Brockett. Input-Out Hamiltonian Systems on Matrix Spaces:

Motivated by problems in neuroscience, we investigate the input-output properties of a class of Hamiltonian systems whose characteristic responses are pulse-like. In order to get an effective theory which allows one to understand the interconnectability of such systems, we investigate the structure of input output systems of this type. One way to introduce coordinates for this class of problems was described by Flaschka who associated a matrix flow with the so-called Toda lattice. In this paper we use his representation, and the appropriate symplectic form, to arrive at notion of input-output structure for Hamiltonian systems defined on certain families of matrices. This leads to an input-output structure for the Toda lattice and, in this framework, one has a effective theory of interconnection, network structures, etc. associated with systems for which the primary mode of communication is through pulses.

Andrew Lewis. Potential energy shaping after kinetic energy shaping:

As concerns stabilisation using energy shaping, the shaping of the potential energy is the crucial step, since here one needs to ensure positive-definiteness of the closed-loop potential function. If one also does kinetic energy shaping, it is possible that, after shaping the kinetic energy, one will end up with a situation where potential energy shaping is impossible. This is because compatibility conditions arise in the partial differential equation for potential shaping. In this paper we investigate these compatibility conditions on the problem of potential energy shaping, done post kinetic energy shaping.

J. Baillieul. Remarks on a Simple Control Law for Point Robot Formations with Exponential Complexity:

This paper considers a simple decentralized control law which stabilizes rigid formations of point robots in a way that is consistent with recent work B.D.O. Anderson and a number of others. The feedback law is simple in that it is shown to make parsimonious use of feedback information in a sense that will be made precise, and it is also simple in that it is the obvious one would try to control the relative positions of a group of robots. The law is exponentially complex, however, in the sense that the number of stable equilibria grows exponentially as a function of the number of point robots involved in the formation. Assuming only one formation of the exponentially many possibilities is of interest, we discuss the problem of achieving the desired formation using the feedback law. The paper will also discuss the combinatorial problem of determining all possible formation topologies as well as the critical point theory of the proposed control law.

A.M. Bloch, M. Leok, J.E. Marsden, and D. Zenkov. Controlled Lagrangians and Potential Shaping for Stabilization of Discrete Mechanical Systems:

In this paper, the method of controlled Lagrangians for discrete mechanical systems is extended to include potential shaping in order to achieve complete state-space asymptotic stabilization. This study is motivated by the importance of structure-preserving algorithms for numerical simulation of controlled systems. The discrete controlled dynamics is used to construct a real-time model predictive controller for the continuous-time system with piecewise constant control inputs. The theoretical analysis is validated by simulating the discrete cart-pendulum system on an incline.

Keyong Li and Raffaello D'Andrea. Trajectory Design of Autonomous Vehicles Based on Motion Primitives and Heuristic Cost-To-Go Functions:

The task of trajectory design of autonomous vehicles is two-fold. First, it needs to take into account of the intrinsic dynamics of the vehicle, which is sometimes termed local constraints. Second, on a higher level, the designed trajectories must allow the vehicle to achieve some application-specific task. The specification of the task results in the so-called global constraints. Both of these two components of trajectory design are generally nontrivial problems, and very often, they are pursued as two parallel areas. When the results drawn from the two areas are applied in conjunction, the synthesis is usually somewhat arbitrary. In this paper, we assume some optimal control strategy that addresses the vehicle dynamics is available as a set of motion primitives. The trajectories that achieve the task are determined solely through the primitives and do not reference the vehicle dynamics directly. For the higher level, we translate the task into a very special type of cost-to-go function, which is partially specified artificially, and partially determined by an admissibility condition imposed by the set of primitives. The optimality feature of the primitives is formally extended to the final trajectory design. We illustrate our result with the example of a mobile robot retrieving an object. On the other hand, our techniques may also facilitate the learning of task execution.

P. Vishwanada Reddy, Eric W. Justh, and P. S. Krishnaprasad. Motion Camouflage in Three Dimensions:

In nature, certain pursuit strategies are observed in visual insects engaged in territorial battles and mating chase that can be interpreted as providing a type of masking or camouflage of a pursuing insect with respect to a background, as viewed by the pursued insect via a visual system which is not very sensitive to looming cues. The underlying geometry has been investigated in a recent paper [1], with a focus on the two-dimensional setting. Using methods from the theory of moving frames, specifically the idea of a relatively parallel adapted frame, we show that a similar analysis can be performed in three dimensions. This paper discusses feedback control laws that arise in the three dimensional setting and connections to biological observations.

[1] E.W. Justh and P.S. Krishnaprasad (2005). "Steering laws for motion camouflage," submitted for publication, (see also arXiv:math.OC/0508023v1).