MATH 170C HOMEWORK 6

(1) Write a MATLAB routine to solve an initial-value problem $x' = f(t, x)$ with $x(t_0) = x_0$ on an interval $a \leq t \leq b$ using the fourth-order Runge–Kutta method with stepsize h. This function should be written so that it can be called in MATLAB by typing:

```
[x,t] = RK4(@f,x0,a,b,h)
```

(a) Consider the following initial-value problem,

$$x' = \lambda x + \cos t - \lambda \sin t, \qquad x(0) = 0$$

Compare your numerical solution (from RK4) to the exact solution on the interval $[0, 5]$ for different values of $\lambda = 5, -5, -10$, and stepsize $h = 0.01$. What effect does λ have on the numerical accuracy?

(2) Write a MATLAB routine to solve an initial-value problem $x' = f(t, x)$ with $x(t_0) = x_0$ on an interval $a \leq t \leq b$ using the fourth-order Adams-Moulton method with stepsize h. This function should be written so that it can be called in MATLAB by typing:

```
[x,t] = AM4(@f,x0,a,b,h, TOL, MaxIters)
```

Use the RK4 method you implemented earlier to obtain the starting values, and use a fixed point iteration to solve the nonlinear equation.

(a) Consider the following initial-value problem,

$$x' = -2tx^2, \qquad x(0) = 1$$

Compute the solution on the interval $[0, 1]$ with stepsize $h = 0.25$ and compare your results with the exact solution $x(t) = 1/(1 + t^2)$.