

MATH 170C ASSIGNMENT 3

§7.1, 7: Derive the following two formulas for approximating the third derivative. Find their error terms. Which formula is more accurate?

$$f'''(x) \approx \frac{1}{h^3} [f(x+3h) - 3f(x+2h) + 3f(x+h) - f(x)]$$

$$f'''(x) \approx \frac{1}{2h^3} [f(x+2h) - 2f(x+h) + 2f(x-h) - f(x-2h)]$$

§7.1, 14: Using Taylor series, derive the error term for the approximation

$$f'(x) \approx \frac{1}{2h} [-3f(x) + 4f(x+h) - f(x+2h)]$$

§7.1, 15: Derive a numerical differentiation formula of order $\mathcal{O}(h^4)$ by applying Richardson extrapolation to

$$f'(x) \approx \frac{1}{2h} [f(x+h) - f(x-h)] - \frac{h^2}{6} f'''(x) - \frac{h^4}{120} f^{(5)}(x)$$

§7.1, 17: Establish a formula of the form

$$f''_n \approx \frac{1}{h^2} [Af_{n+3} + Bf_{n+2} + Cf_{n+1} + Df_n]$$

Here $f_{n+i} = f(x_n + ih)$.