MATH 170C ASSIGNMENT 1

§7.1, 10: Show how to use Richardson extrapolation employing x_n and x_{n^2} if

$$L = x_n + a_1 n^{-1} + a_2 n^{-2} + a_3 n^{-3} + \cdots$$

§7.1, 12: Show how to use Richardson extrapolation if

$$L = \varphi(h) + a_1 h + a_3 h^3 + a_5 h^5 + \cdots$$

§7.1, 13: Suppose that $L = \lim_{h\to 0} f(h)$ and that $L - f(h) = c_6 h^6 + c_9 h^9 + \cdots$. What combination of $f(h)$ and $f(h/2)$ should be the best estimate of L?

§7.2, 1: Derive the Newton-Cotes formula for $\int_0^1 f(x)dx$ based on nodes 0, $\frac{1}{3}$, $\frac{2}{3}$, and 1.

§7.2, 4 and 5: Verify that the following formula is exact for polynomials of degree ≤ 4.

$$\int_0^1 f(x)dx \approx \frac{1}{90}\left[7f(0) + 32f\left(\frac{1}{4}\right) + 12f\left(\frac{1}{2}\right) + 32f\left(\frac{3}{4}\right) + 7f(1)\right].$$

(continuation) From the formula above, obtain a formula for $\int_a^b f(x)dx$ that is exact for all polynomials of degree 4. (Apply a suitable change of variables to the integral and the quadrature formula)

§7.2, 8: Find the formula

$$\int_0^1 f(x)dx \approx A_0 f(0) + A_1 f(1)$$

that is exact for all functions of the form $f(x) = ae^x + b\cos(\pi x/2)$.
(Write down conditions for the quadrature formula to be exact for $f_1(x) = e^x$ and $f_2(x) = \cos(\pi x/2)$ and solve for A_0 and A_1.)

§7.2, 10: Use the Lagrange interpolation polynomial to derive the formula of the form

$$\int_0^1 f(x)dx \approx Af\left(\frac{1}{3}\right) + Bf\left(\frac{2}{3}\right).$$

Transform this formula to one for integration over $[a, b]$.