

A New Family of Solutions to the Kepler-Heisenberg Problem

Corey Shanbrom
CSU Sacramento

Abstract

The Kepler-Heisenberg problem is that of determining the motion of a planet around a sun in the Heisenberg group, thought of as a three-dimensional sub-Riemannian manifold. The sub-Riemannian Hamiltonian provides the kinetic energy, and the gravitational potential is given by the fundamental solution to the sub-Laplacian. The dynamics are at least partially integrable, possessing two first integrals as well as a dilational momentum which is conserved by orbits with zero energy. The system is known to admit closed orbits, which all lie within a fundamental integrable subsystem. Here, we present the numerical discovery of a new and beautiful family of solutions.