

69th ANNUAL HIGH SCHOOL HONORS MATHEMATICS CONTEST

April 18th, 2026
on the campus of the
University of California, San Diego

PART II 4 Questions

Welcome to Part II of the contest!

Please print your Name, and School:

Name _____
First Last

School _____

Please do not open the exam until told to do so by the proctor.

EXAMINATION DIRECTIONS:

1. Print (**clearly**) your Name and School on **each page of the contest**.
2. Part II consists of 4 problems, each worth 25 points. These problems are “essay” style questions. You should put all work towards a solution in the space following the problem statement. You should show all work and justify your responses as best you can.
3. Scoring is based on the progress you have made in understanding and solving the problem. The clarity and elegance of your response is an important part of the scoring.
4. You may use the back side of each sheet to continue your solution, but be sure to call the reader’s attention to the back side if you use it. If you run out of space, you may include answers on extra exam booklets provided upon request, but be sure that make the proctors aware of this when turning in your exam.
5. Give this entire exam to a proctor when you have completed the test to your satisfaction.

Please let your coach know if you plan to attend the Awards Banquet on April 29th.

Name: _____

School: _____

Problem 1 Determine all polynomials $P(x)$ of degree $n \geq 1$ such that $xP(x) = (x - n)P(x + 1)$, for every real number x .

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Problem 2 Consider a rectangle $ABCD$. Let α denote the angle BAC . Let B_1 and D_1 be points distinct from A on the segments AB and AD , respectively. Denote by C_1 the intersection of B_1D_1 and AC . Prove that

$$\frac{1}{|AC_1|} = \frac{\cos \alpha}{|AB_1|} + \frac{\sin \alpha}{|AD_1|}.$$

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Problem 3 *Suppose that four circles are pairwise orthogonal to each other. Show that the centers of these circles form a triangle and its orthic center (the orthic center of a triangle is the intersection of its altitudes).*

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Problem 4 Let T_n be the maximum number of triangles that can be placed in three dimensional space so that no two of these triangles overlap except on corners or edges and all together there are at most n points used as vertices of these triangles. Prove that for $n \geq 3$:

$$(n - 3)^2/2 \leq T_n \leq n^2.$$