

69thANNUAL HIGH SCHOOL HONORS MATHEMATICS CONTEST

April 18th, 2026
on the campus of the
University of California, San Diego

PART I 25 Questions

Welcome to the contest! Please do not open the exam until told do so by the proctor.

EXAMINATION DIRECTIONS:

1. Print (**clearly**) your Name and school at the top of **each page of the contest**.
2. Calculators or other electronic devices may **not** be used.
3. There are 25 multiple-choice questions on Part I. You have 90 minutes for Part I.
4. Clearly indicate your answer in the bubble for each problem. Fill in the bubble, or put a clear X, or a checkmark; just make sure your selection is clear.

Scoring

+4 points for a correct answer.
0 points for no answer.
–1 points for an incorrect answer.

Good Luck!

There will be a 15 minute break after Part I before proceeding to Part II.

Please let your coach know if you plan to attend the Awards Banquet on April 29th.

Name: _____

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Problem 1 Anna wants to count the number of Beryllium atoms in the universe. She knows that there are 10^{10} Helium atoms for every Beryllium atom, and furthermore, that the number of Beryllium and Helium atoms together make up 4% of all atoms. Assuming that there are $10^{78} + 10^{68}$ atoms in the universe, how many Beryllium atoms are there?

- ☐ (A) 2.5×10^{66}
- ☐ (B) 4×10^{66}
- ☐ (C) $4 \times 10^{67} + 4 \times 10^{57}$
- ☐ (D) $2.5 \times 10^{68} + 2.5 \times 10^{56}$
- ☐ (E) 4×10^{68}

Problem 2 A classroom contains 1 teacher and 7 students. If the teacher is talking, no students may talk. If the teacher is silent, any subset of students may talk simultaneously. How many valid subsets of talking people are possible?

- ☐ (A) 64
- ☐ (B) 65
- ☐ (C) 127
- ☐ (D) 128
- ☐ (E) 129

Problem 3 An equilateral triangle is inscribed in a circle, which is inscribed in a square. What is the ratio of the triangle's side length to the square's side length?

- ☐ (A) $1/2$
- ☐ (B) $\sqrt{2}/2$
- ☐ (C) $\sqrt{3}/2$
- ☐ (D) $\pi/2$
- ☐ (E) $\sqrt{3}$

Problem 4 All four faces of a tetrahedron have the same perimeter. What is the maximum number of distinct side lengths that it could have?

- ☐ (A) 1
- ☐ (B) 2
- ☐ (C) 3
- ☐ (D) 4
- ☐ (E) 6

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Problem 5 Let x_1, x_2, x_3 be the roots of the polynomial $x^3 - x^2 - 4x + 2 = 0$. What is the value of $\frac{1}{x_1} + \frac{1}{x_2} + \frac{1}{x_3}$?

- ☐ (A) -4
- ☐ (B) 0
- ☐ (C) $\frac{1}{2}$
- ☐ (D) $\sqrt{3}$
- ☐ (E) 2

Problem 6 If P is a polynomial of degree $n \geq 1$ such that $P(1) = P(2) = \cdots = P(n) = 0$ and $P(n+1) = 1$, then $P(0)$ is equal to

- ☐ (A) -1
- ☐ (B) $(-1)^n$
- ☐ (C) 2^n
- ☐ (D) $n!$
- ☐ (E) $(n+1)!$

Problem 7 Let $p_1, p_2, \dots, p_n$ be distinct odd primes and define $a = p_1 p_2 \cdots p_n$. What is the number of distinct pairs of integers (x, y) which satisfy $x^2 + a^2 = y^2$?

- ☐ (A) $n^2 + 1$
- ☐ (B) $n^3 + n^2 - 1$
- ☐ (C) $2^n + 1$
- ☐ (D) $2 \cdot 3^n$
- ☐ (E) $4^n + 2$

Problem 8 Which of these is the median?

- ☐ (A) 20^{26}
- ☐ (B) 62^{20}
- ☐ (C) $(6^2)^{20}$
- ☐ (D) $(2^2)^{60}$
- ☐ (E) 20^{6^2}

Problem 9 What is the smallest possible value of $\sqrt{x^2 + y^2} + \sqrt{(x-3)^2 + (y-4)^2}$, where x and y range over all real numbers?

- ☐ (A) $\sqrt{5}$
- ☐ (B) $\frac{7}{2}$
- ☐ (C) 4
- ☐ (D) 5
- ☐ (E) 8

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Problem 10 What is $3 + 3^2 + 3^3 + \dots + 3^{2026} \pmod{2026}$?

- ☐ (A) 0
- ☐ (B) 2
- ☐ (C) 3
- ☐ (D) 12
- ☐ (E) 38

Problem 11 A convex polygon P has perimeter 2026. Let S be the set of points exterior to P but within one unit distance of it. What is the area of S ?

- ☐ (A) 2026
- ☐ (B) $2026 + \pi$
- ☐ (C) $2026 + (\text{Number of sides of } S)\pi$
- ☐ (D) 4052
- ☐ (E) $4052 + \pi$

Problem 12 A bag contains 6 red marbles and 6 blue marbles. You draw marbles one at a time (without replacement) until you have drawn 6 marbles of the same color. How many distinct sequences of colors can you form?

- ☐ (A) 84
- ☐ (B) 210
- ☐ (C) 462
- ☐ (D) 792
- ☐ (E) 924

Problem 13 What is the smallest value of $x + y + \frac{1}{x^2y}$, where x and y range over all positive real numbers?

- ☐ (A) $\frac{7}{4}$
- ☐ (B) 2
- ☐ (C) $\sqrt{5}$
- ☐ (D) $2\sqrt{2}$
- ☐ (E) 4

Problem 14 How many real solutions does the equation $2026^x = 2026x$ have?

- ☐ (A) 0
- ☐ (B) 1
- ☐ (C) 2
- ☐ (D) 3
- ☐ (E) Infinitely many

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Problem 15 Let n be a positive integer. The number of bijections $f : \{1, 2, \dots, 2n\} \rightarrow \{1, 2, \dots, 2n\}$ satisfying $f(k) + f(2n + 1 - k) = 2n + 1$, for every $k \in \{1, 2, \dots, 2n\}$ is equal to:

- ☐ (A) 2^n
- ☐ (B) $\binom{2n}{n}$
- ☐ (C) $n!$
- ☐ (D) $2^n n!$
- ☐ (E) 2^{n^2}

Problem 16 Triangle ABC has $AB = 7$, $BC = 11$, $CA = 12$. Circles are drawn centered at each of A , B , and C so that each pair of these circles meet at right angles. What is the radius of the circle centered at A ?

- ☐ (A) 5
- ☐ (B) $\sqrt{30}$
- ☐ (C) $4\sqrt{2}$
- ☐ (D) 6
- ☐ (E) 7

Problem 17 For how many values of $n \geq 3$ does there exist a tessellation of the plane by regular n -gons such that the intersection of any two polygons is either empty or a line segment (but not just a single vertex)?

- ☐ (A) 0
- ☐ (B) 1
- ☐ (C) 2
- ☐ (D) 3
- ☐ (E) 4

Problem 18 A circle of radius 2 centered on the positive x -axis is tangent to the hyperbola $x^2 - y^2 = 1$ at exactly two points. What is the x -coordinate of these points?

- ☐ (A) $\sqrt{3}/2$
- ☐ (B) $\sqrt{5}/2$
- ☐ (C) $\sqrt{2}$
- ☐ (D) $\sqrt{10}/2$
- ☐ (E) $\sqrt{5}$

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Problem 19 Let $x_1, x_2, \dots, x_{2026}$ be the roots of the polynomial $x^{2026} + 4x + 1 = 0$. What is the value of $(x_1^2 + 1)(x_2^2 + 1) \cdots (x_{2026}^2 + 1)$?

- ☐ (A) $-8i$
- ☐ (B) 0
- ☐ (C) $\sqrt{2}$
- ☐ (D) 4
- ☐ (E) 16

Problem 20 Let $S = \{(x, y) : x, y \in \{1, 2, \dots, 10\}\}$ be a set of points in the plane. How many ways can you pick a pair of points in S so that the distance between these points is a positive integer?

- ☐ (A) 1068
- ☐ (B) 1076
- ☐ (C) 1084
- ☐ (D) 1092
- ☐ (E) 1100

Problem 21 Given an integer n we define its reverse by first writing n in binary (with no leading 0's), and then reversing the order of the digits (so that the resulting number may have a leading 0). How many integers in $\{1, 2, \dots, 2026\}$ are larger than their reverse?

- ☐ (A) 1468
- ☐ (B) 1469
- ☐ (C) 1470
- ☐ (D) 1471
- ☐ (E) 1472

Problem 22 An integer n is picked uniformly at random from $\{1, 2, \dots, 342\}$. A uniform random divisor d of n is then selected. What is the probability that d is a multiple of 7?

- ☐ (A) $4/57$
- ☐ (B) $25/342$
- ☐ (C) $51/684$
- ☐ (D) $103/1372$
- ☐ (E) $1/7$

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Problem 23 Define a sequence as follows: $A_1 = 1$ and

$$A_n = \begin{cases} -A_{n-1} + A_{(n-1)/2} & n \text{ odd} \\ -A_{n-1} & n \text{ even} \end{cases}$$

For $n > 1$. Which is the largest

- ☐ (A) A_{2019}
- ☐ (B) A_{2021}
- ☐ (C) A_{2023}
- ☐ (D) A_{2025}
- ☐ (E) A_{2027}

Problem 24 What is the greatest common divisor of all numbers of the form $n^{20} - n^{26}$ for integers n ?

- ☐ (A) 28
- ☐ (B) 63
- ☐ (C) 168
- ☐ (D) 252
- ☐ (E) 504

Problem 25 Let S be the set of fractions p/q that are in reduced form with $0 < p < q \leq 100$. What is

$$\sum_{p/q \in S} (p/q)[100/q]$$

where $[x]$ denotes the greatest integer less than or equal to x .

- ☐ (A) 2472
- ☐ (B) 2473
- ☐ (C) 2474
- ☐ (D) 2475
- ☐ (E) 2476