66th ANNUAL
HIGH SCHOOL HONORS MATHEMATICS CONTEST

April 22, 2023
on the campus of the
University of California, San Diego

PART II
4 Questions

Welcome to Part II of the contest!

Please print your Name, School, and Contest ID number:

Name ______________________________________
First   Last

School ______________________________________

Contest ID number ______________________________

Please do not open the exam until told do so by the proctor.

EXAMINATION DIRECTIONS:

1. Print (**clearly**) your Name and Contest ID number on **each page of the contest**.
2. Part II consists of 4 problems, each worth 25 points. These problems are "essay" style questions. You should put all work towards a solution in the space following the problem statement. You should show all work and justify your responses as best you can.
3. Scoring is based on the progress you have made in understanding and solving the problem. The clarity and elegance of your response is an important part of the scoring. You may use the back side of each sheet to continue your solution, but be sure to call the reader's attention to the back side if you use it.
4. Give this entire exam to a proctor when you have completed the test to your satisfaction.

Please let your coach know if you plan to attend the Awards Banquet on Tuesday, May 2, 6:00–8:30pm.

Name: ID Number:

Problem 1 *Let $f, g : \mathbb{R} \to \mathbb{R}$ be two functions such that*

$$f(x)f(y) = [g(x+y)], \quad f(0) = 1.$$

Show that $f(x) = 1$ for all $x \in \mathbb{R}$. Here $[x]$ denotes the integer part of x.

Name: ID Number:

Problem 2 *Let S be a set of 132 positive integers such that $3 \in S$ and $97 \in S$. Show that there exist three distinct elements $a, b, c \in S$ and $[a, b] \leq c$. Here, $[a, b]$ denotes the least common multiple of a and b.*

Name: ID Number:

Problem 3 *Let $n \geq 3$. Prove that the maximum number of line segments which can be drawn between the vertices of a regular n-gon in the plane without having two disjoint line segments is n.*

Problem 4 *Prove or disprove: For any positive integer* N *there exists a set* $S \subset \{1, 2, \ldots, N^2\}$ *with* $|S| = N$ *so that there is no integer* n *that can be written as* $n = x + y$ *with* $x, y \in S$ *in more than* 2023 *different ways.*