65th ANNUAL
HIGH SCHOOL HONORS MATHEMATICS CONTEST

April 16, 2022
on the campus of the
University of California, San Diego

PART II
4 Questions

Welcome to Part II of the contest!

Please print your Name, School, and Contest ID number:

Name ____
First Last

School ____

Contest ID number ____________________________

Please do not open the exam until told do so by the proctor.

EXAMINATION DIRECTIONS:

1. Print (**clearly**) your Name and Contest ID number on **each page of the contest**.
2. Part II consists of 4 problems, each worth 25 points. These problems are "essay" style questions. You should put all work towards a solution in the space following the problem statement. You should show all work and justify your responses as best you can.
3. Scoring is based on the progress you have made in understanding and solving the problem. The clarity and elegance of your response is an important part of the scoring. You may use the back side of each sheet to continue your solution, but be sure to call the reader's attention to the back side if you use it.
4. Give this entire exam to a proctor when you have completed the test to your satisfaction.

Name: ID Number:

Problem 1 *Solve the equation below for x:*

$$\cos(\pi \log_3(x+6)) \cos(\pi \log_3(x-2)) = 1.$$

Problem 2 *Let BD be a fixed line segment. Find the geometric locus (set of all points) A such that there exists an isosceles triangle ABC for which $AB = AC$ so that BD is the median of the edge AC.*

Problem 3 *Suppose we have an infinite sequence of numbers* $a_0, a_1, \ldots, a_n, \ldots$*, all between* 0 *and* 1*, such that for any* $n \geq 0$*,*

$$a_{n+2} - 2a_{n+1} + a_n \geq 0 \ .$$

Show that the sequence must be decreasing, and that $0 \leq a_n - a_{n+1} \leq \frac{1}{n+1}$ *for all* n*.*

Name: ID Number:

Problem 4 *Suppose that $z_1, z_2, \ldots, z_n$ are complex numbers so that for every integer k from 1 to n we have that $\sum_{i=1}^{n} z_i^k = 2n$. What is $\sum_{i=1}^{n} z_i^{n+1}$?*