64th ANNUAL
HIGH SCHOOL HONORS MATHEMATICS CONTEST

April 24-25, 2021

PART II
4 Questions

Welcome to Part II of the contest!

Please do not open the exam until told do so by the proctor.

EXAMINATION DIRECTIONS:

1. Calculators or other electronic devices may **not** be used. If you cannot conveniently work on a physical copy of the exam, you *may* use a computer but **only** to view the exam and type your answers. You may not use an internet connection or any other programs on your computer.

2. You may use blank scratch paper, pens, pencils, compass and straight edge.

3. Part II consists of 4 problems, each worth 25 points. These problems are "essay" style questions. You should put all work towards a solution in the space following the problem statement. You should show all work and justify your responses as best you can.

4. Scoring is based on the progress you have made in understanding and solving the problem. The clarity and elegance of your response is an important part of the scoring.

Please mark your calendars for the awards ceremony (held over zoom meeting) Saturday, May 1, 6:30–8:30pm.

Problem 1 *Matt has a biased coin that is more likely to come up heads than tails. He flips this coin n times and counts the number of tails. Show that this number is more likely to be even than it is to be odd.*

Problem 2 *Let $n \geq 2$ be a positive integer and X be a set containing n^2 consecutive numbers. Let A be a subset of X with n elements. Show that $X \setminus A$ contains at least one arithmetic progression with n elements.*

Problem 3 *Let $n > 0$ be an integer. It is known that the difference d of two divisors of 55^n is a power of 2. Show that $d = 4$.*

Problem 4 *Let C be a set of n points on a circle in the plane. Prove that amongst any set of $n+1$ line segments between points in C, there exist two geometrically disjoint line segments, but that n segments are not sufficient.*