61st ANNUAL
HIGH SCHOOL HONORS MATHEMATICS CONTEST

April 21, 2018
on the campus of the
University of California, San Diego

PART II
4 Questions

Welcome to Part II of the contest!

Please print your Name, School, and Contest ID number:

Name ____
First   Last

School ____

3-digit Contest ID number ______________________________

Please do not open the exam until told do so by the proctor.

EXAMINATION DIRECTIONS:

1. Print (**clearly**) your Name and Contest ID number on **each page of the contest**.
2. Part II consists of 4 problems, each worth 25 points. These problems are "essay" style questions. You should put all work towards a solution in the space following the problem statement. You should show all work and justify your responses as best you can.
3. Scoring is based on the progress you have made in understanding and solving the problem. The clarity and elegance of your response is an important part of the scoring. You may use the back side of each sheet to continue your solution, but be sure to call the reader's attention to the back side if you use it.
4. Give this entire exam to a proctor when you have completed the test to your satisfaction.

Please let your coach know if you plan to attend the Awards Banquet on Wednesday, May 2, 6:00–8:30pm in the UCSD Faculty Club.

Name: ID Number:

Problem 1 *A convex polygon with n sides has all angles equal to 150 degrees, with the possible exception of one angle. List all the possible values of n.*

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Problem 2 *Let* $\mathbb{N} = \{1, 2, 3, ...\}$ *be the set of positive integers.*

(a) *Let* $f : \mathbb{N} \to \mathbb{N}$ *be a strictly increasing function such that* $f(f(n)) = n + 2018$*, for any* $n \in \mathbb{N}$*. Prove that* $f(n) = n + 1009$*, for every* $n \in \mathbb{N}$*.*

(b) *Give an example of function* $f : \mathbb{N} \to \mathbb{N}$ *which is not strictly increasing function and satisfies* $f(f(n)) = n + 2018$*, for every* $n \in \mathbb{N}$*.*

Name: ID Number:

Problem 3 *Let A, B, C be points on a circle of radius 1 such that $|AB|^2 + |BC|^2 + |CA|^2 = 8$. Prove that ABC is a right triangle.*

Problem 4 *Define the Collatz sequence starting with a number m to be the sequence of integers defined by $a_1 = m$ and for $n \geq 1$,*

$$a_{n+1} = \begin{cases} a_n/2 & \textit{if } a_n \textit{ is even.} \\ 3a_n + 1 & \textit{if } a_n \textit{ is odd.} \end{cases}$$

Prove that there are integers n, m with $2^n > m > 0$ so that the Collatz sequence starting from m contains terms larger than 3^{n+2018}.