60th ANNUAL
HIGH SCHOOL HONORS MATHEMATICS CONTEST

April 22, 2017
on the campus of the
University of California, San Diego

PART II
4 Questions

Welcome to Part II of the contest!

Please print your Name, School, and Contest ID number:

Name ____
First Last

School ____

3-digit Contest ID number ______________________________

Please do not open the exam until told do so by the proctor.

EXAMINATION DIRECTIONS:

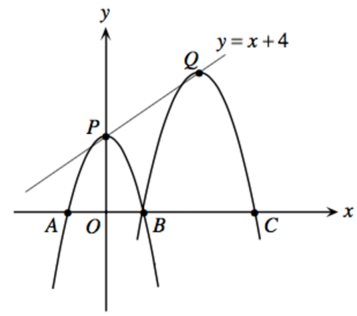
1. Part II consists of 4 problems, each worth 25 points. These problems are "essay" style questions. You should put all work towards a solution in the space following the problem statement. You should show all work and justify your responses as best you can.

2. Scoring is based on the progress you have made in understanding and solving the problem. The clarity and elegance of your response is an important part of the scoring. You may use the back side of each sheet to continue your solution, but be sure to call the reader's attention to the back side if you use it.

3. Give this entire exam to a proctor when you have completed the test to your satisfaction.

Please let your coach know if you plan to attend the Awards Dinner on Wednesday, May 3, 6:00–8:30pm in the UCSD Faculty Club.

1. The parabola with equation $y = 4 - x^2$ has vertex P and intersects the x-axis at A and B. The parabola is translated from its original position so that its vertex moves along the line $y = x + 4$ to the point Q. In this position, the parabola intersects the x-axis at B and C. Determine the coordinates of C.



2. 15 pairwise coprime integers are chosen from the set $\{2, \ldots, 2017\}$. Show that at least one prime number was chosen.

3. Let $a_1 < a_2 < \cdots < a_n$ be n real numbers such that the set

$$A = \{a_j - a_i \mid 1 \leq i < j \leq n\}$$

has exactly $n-1$ elements. Prove that $a_1, a_2, ..., a_n$ form an arithmetic progression.

4. Let S be a region in space given as a finite union of unit cubes whose corners have integer coordinates. Let A be the projection of S onto the (x, y)-plane, B the projection onto the (y, z)-plane and C the projection onto the (x, z)-plane. Prove that

$$\mathrm{Vol}(S) \leq \sqrt{\mathrm{Area}(A)\mathrm{Area}(B)\mathrm{Area}(C)}.$$