

59th ANNUAL HIGH SCHOOL HONORS MATHEMATICS CONTEST

April 16, 2016
on the campus of the
University of California, San Diego

PART II 4 Questions

Welcome to Part II of the contest!

Please print your Name, School, and Contest ID number:

Name _____
First Last

School _____

3-digit Contest ID number _____

Please do not open the exam until told do so by the proctor.

EXAMINATION DIRECTIONS:

1. Part II consists of 4 problems, each worth 25 points. These problems are “essay” style questions. You should put all work towards a solution in the space following the problem statement. You should show all work and justify your responses as best you can.
2. Scoring is based on the progress you have made in understanding and solving the problem. The clarity and elegance of your response is an important part of the scoring. You may use the back side of each sheet to continue your solution, but be sure to call the reader’s attention to the back side if you use it.
3. Give this entire exam to a proctor when you have completed the test to your satisfaction.

Please let your coach know if you plan to attend the Awards Dinner on Wednesday, April 27, 6:00–8:30pm in the UCSD Faculty Club.

1. A fair coin is tossed 10 times. What is the probability that it comes up heads 5 times in a row, but not 6 times in a row?

2. Let R be a rectangle whose sides have lengths 2 and 3. Choose any four points inside R . Prove that there exist two of these points whose distance from each other is less than $\sqrt{5}$.

3. What is the largest positive integer n so that n is not the area of a union of two squares with corners on lattice points and sides parallel to the x - and y -axes? For example, 12 is the area of the following union of squares of side lengths 2 and 3.



4. Let $p_1 = 2$ and define p_{n+1} to be the largest prime divisor of $1 + p_1 p_2 \dots p_n$. Is 11 a term in the sequence $\{p_n\}$?