# 58th ANNUAL
# HIGH SCHOOL HONORS MATHEMATICS CONTEST

April 18, 2015
on the campus of the
University of California, San Diego

**PART I**
**ANSWER KEY**

| | |
|---|---|
| 1. (D) | 14. (B) |
| 2. (A) | 15. (A) |
| 3. (E) | 16. (C) |
| 4. (B) | 17. (A) |
| 5. (C) | 18. (D) |
| 6. (B) | 19. (A) |
| 7. (A) | 20. (C) |
| 8. (E) | 21. (D) |
| 9. (E) | 22. (C) |
| 10. (A) | 23. (E) |
| 11. (E) | 24. (D) |
| 12. (D) | 25. (B) |
| 13. (C) | |

(A)$\times 6$ (B)$\times 4$ (C)$\times 5$ (D)$\times 5$ (E)$\times 5$

You may take these exam questions with you after you are done. You may write on this exam and use it to discuss your results outside the room after completion of the exam.

1. An engineer said she could finish a highway section in three days with her present supply of a certain type of machine. However, with three more of these machines the job could be done in two days. Assume that the machines all work at the same rate. What number of days would take it to do the job with one machine?

   (A) $6$ (B) $12$ (C) $15$ (D) $18$ (E) $36$

2. Let $x = \sqrt{7+2\sqrt{6}} + \sqrt{7-2\sqrt{6}}$. Which of the following is an alternative expression for $x$?

   (A) $\sqrt{24}$ (B) $\frac{12}{\sqrt{7+2\sqrt{6}}-\sqrt{7-2\sqrt{6}}}$ (C) $2\sqrt{7}$ (D) $6$ (E) $2\sqrt{7}-\sqrt{6}$

3. Earthquakes generate p-waves and s-waves from the epicenter of the earthquake. The p-waves travel faster than the s-waves. As a methamtical model, assume that the p-waves travel 1.5 km/s and the s-waves travel 1 km/s. Assume that, at a seismic observation station, the p-waves arrive 1 minute and 30 seconds before the s-waves. What is the approximate distance from the epicenter of the earthquake to the seismic observation station?

   (A) 90km (B) 100km (C) 135km (D) 27km (E) 270km

4. How many integers between 1 and 1000 have 3 as at least one digit?

   (A) 100 (B) ~~162~~ 271 (C) 190 (D) 290 (E) 333

5. What is the number of roots of $\frac{15}{x^2-4} - \frac{2}{x-2} = 1$?

   (A) $0$ (B) $1$ (C) $2$ (D) $3$ (E) $4$

6. Consider the following combination of given parts and the type of a triangle. What combination does *not* determine the triangle?

   (A) vertex angle and the base; isosceles triangle

   (B) base angle and vertex angle; isosceles triangle

   (C) the radius of the circumscribed circle; equilateral triangle

   (D) one leg and the radius of the inscribed circle; right triangle

   (E) two angles and a side opposite to one of them; scalene triangle

7. Consider the equation $\sqrt{x+4} - \sqrt{x-3} + 1 = 0$. How many real roots does the equation have?

   (A) $0$ (B) $1$ (C) $2$ (D) $3$ (E) $4$

8. A thick, flexible cable is wrapped by a machine very neatly and uniformly about a spool. The middle of the spool is a two-foot long cylinder. The diameter of the cylinder is two feet. It is known that when the cable has wrapped around the cylinder enough times to form a cylinder of diameter four feet, the length of that part of the cable is 1200 feet. Approximately what is the length of the cable that will fill up the spool, that is, will form a segment of a cylinder that is six feet in diameter and two feet long?

   (A) $4000$ (B) $2400$ (C) $12000$ (D) $3600$ (E) $3200$

9. Let the letters $w, x, y, z$ represent real numbers. Which of the five expressions below is equal to $(xz - yw)^2 + (xw + yz)^2$?

   (A) $(xz + yw)^2 + (xw + yz)^2$
   (B) $(xz - yw)^2 + (xw - yz)^2$
   (C) $(xz - yw + xw + yz)^2$
   (D) $(xz - yw + xw - yz)^2$
   (E) $(x^2 + y^2)(z^2 + w^2)$

10. Consider the following frustum pyramid. The horizontal base is a square 6 by 6. The upper base is a horizontal square 4 by 4. In each of the bases there is a pair of sides that are north south and another that is east west. The vertical distance between the bases is 2. What is the volume of the frustum?

    (A) $\frac{152}{3}$ (B) $\frac{64}{3}$ (C) $72$ (D) $\frac{280}{3}$ (E) $64$

11. Let $S = \sqrt{2 + \sqrt{2 + \sqrt{2 + \ldots}}}$. What is $S$?

(A) $\sqrt{2}$ (B) $\frac{1+\sqrt{5}}{2}$ (C) $\frac{1+\sqrt{13}}{2}$ (D) $-1$ (E) $2$

12. Consider a quadratic equation $ax^2+bx+c=0$, where $a \neq 0$. What condition implies that the roots of this equation are reciprocals of each other?

(A) $ab = c$ (B) $a = b$ (C) $a = bc$ (D) $c = a$ (E) $c = b$

13. A fair six-sided die is case six times. What is the probability that six shows up at least once?

(A) $\frac{15{,}625}{46{,}656}$ (B) $\frac{15{,}625}{31{,}031}$ (C) $\frac{31{,}031}{46{,}656}$ (D) $\frac{625}{1{,}296}$ (E) $\frac{671}{1{,}296}$

14. In a certain fishing party there are five people. There are five groups of four of the people. The sums of the ages of members of each of these four-person groups are $142$, $136$, $130$, $128$, and $124$. What is the age of the oldest person in the fishing party?

(A) $23$ (B) $41$ (C) $45$ (D) $35$ (E) $48$

15. What is the value of $\sin(15^\circ) + \cos(15^\circ)$?

(A) $\frac{\sqrt{6}}{2}$ (B) $\frac{1}{2}$ (C) $\frac{\sqrt{3}}{2}$ (D) $\sqrt{2}$ (E) $1$

16. Consider the integers **1** 2 **3** 3 4 **6** 7 8 9 **10** 11 12 13 14 **15** 16 17 18 19 20 **21** ... Let $f(n)$ be the sum of the first $n$ **bold** numbers. For example $f(1) = 1$, $f(2) = 4$, $f(3) = 10$, $f(4) = 20$, and so forth. What is the value of $f(30)$?

(A) $2,024$ (B) $3,276$ (C) $4,960$ (D) $7,140$ (E) $9,880$

17. What is the sum of the first 100 prime numbers?

(A) $24,133$ (B) $25,126$ (C) $4,194$ (D) $987$ (E) $512$

18. Consider a right triangle with legs $a$ and $b$. Let $x$ be the length of the altitude drawn from the right angle to the hypotenuse. What is the relation among $x$, $a$, and $b$?

(A) $ab = x^2$ (B) $\frac{1}{a} + \frac{1}{b} = \frac{1}{x}$ (C) $a^2 + b^2 = 2x^2$ (D) $\frac{1}{x^2} = \frac{1}{a^2} + \frac{1}{b^2}$ (E) $\frac{1}{x} = \frac{b}{a}$

19. What is the rightmost digit of $7^{888}$?

(A) 1 (B) 9 (C) 7 (D) 5 (E) 3

20. What are the number of consecutive zeros to the left of the decimal place in $2345!$?

(A) 469 (B) 593 (C) 583 (D) 563 (E) 367

21. What is the largest possible area of the intersection of two unit squares whose sides meet at 45 degree angles?

(A) $1/2$ (B) $\sqrt{2} - 1$ (C) $1/\sqrt{2}$ (D) $2\sqrt{2} - 2$ (E) $4\sqrt{2} - 5$

22. Let $n$ be a positive integer. In how many ways can $n$ be written as the sum of three non-negative integers.

(A) $\frac{n(n+1)}{2}$ (B) $\frac{n(n+1)(n+2)}{6}$ (C) $\frac{(n+1)(n+2)}{2}$ (D) $n^2$ (E) $2^n$

23. Three six-sided dice are rolled. Which of the following products is most likely?

(A) 36 (B) 30 (C) 50 (D) 15 24

24. Let $x, y, z$ be real numbers satisfying the following equations.

$$x + yz = 1$$
$$y + xz = 1$$
$$z + xy = 1$$

Which of the following most accurately describes the number of real solutions $(x, y, z)$?

(A) 0 (B) 3 (C) 6 (D) finitely many, but not 0, 3, or 6 (E) infinitely many

25. Let $n$ be a positive integer. How many functions $f : \{1, 2, ..., n\} \to \{1, 2, ..., n\}$ satisfy $f(x + 1) - f(x) \in \{0, 1\}$, for every $x \in \{1, 2, ..., n - 1\}$?

(A) $2^{n+1}$ (B) $(n + 1)2^{n-2}$ (C) $n^2$ (D) $n2^{n+1}$ (E) $n^n$