Midterm I Practice Problems

Midterm II will cover: Section 4.4, 4.5, 4.6, 4.7, 7.2, 7.3, 7.4, 7.5, 7.6. The following topics are particularly important:

Solve Caychy-Euler equation;
Use the method of undetermined coefficients to solve nonhomogeneous equations;
Use the method of variation of parameters to solve nonhomogeneous equations;
Compute Laplace transform;
Compute the inverse Laplace transform;
Solve differential equation using Laplace transform;
Laplace transform of discontinuous functions.

1. Solve the initial value problem \( x^2y'' - 2xy' + 2y = 0 \), \( y(1) = 2 \), \( y'(1) = 3 \).

2. Use the method of undetermined coefficients to solve the following nonhomogeneous equation. Find the general solution.

\[ y'' - 2y' - 3y = 2e^{3t}. \]

3. Use variation of parameters to solve the following nonhomogeneous equation. Find the general solution.

\[ y'' + 4y = \sec(2t) \]

4. Determine

\[ \mathcal{L}^{-1}\left\{ \frac{3s^2 + 5s + 3}{s^4 + s^3} \right\}. \]

5. Express the given function using window and step functions and compute its Laplace transform.

\[ f(t) = \begin{cases} 0, & t \in [0, \pi/2), \\ \sin t, & t \in [\pi/2, \infty). \end{cases} \]

6. Solve the given initial value problem using the method of Laplace transforms. Sketch the graph of the solution.

\[ y'' + y = u(t - 2) - u(t - 4), \quad y(0) = 1, \, y'(0) = 0. \]