Midterm II Practice Problems

Midterm II will cover sections 87-94.

1. For each of the following integrals, sketch a contour that can be used to evaluate the integral by residues, and give the analytic function $f(z)$ that you would integrate. If the function $f(z)$ involves log function, specify which branch you take. **You do not need to do any computation**.

   (a) $\displaystyle\int_0^\infty \frac{x \sin x}{x^4+1} dx.$

   (b) $\displaystyle\int_0^\infty \frac{\ln x}{x^4+1} dx.$

   (c) $\displaystyle\int_0^{2\pi} \frac{1}{(\cos\theta)^4 + (\sin\theta)^4} d\theta.$

2. Let $0 < \rho < 1 < R$, and let C_ρ denote the upper semicircle $|z| = \rho$ (oriented clockwise) and C_R denote the upper semicircle $|z| = R$ (oriented counterclockwise). Compute the following limits.

   (a) $\displaystyle\lim_{R\to\infty} \int_{C_R} \frac{ze^{iz}}{z^2+1} dz.$

   (b) $\displaystyle\lim_{\rho\to 0} \int_{C_\rho} \frac{1}{z(z^2+1)} dz.$

3. Let $f(z) = \dfrac{(4z+i)^2}{(2z-1)^2(z+5)^9}$. Let C denote the unit circle $|z| = 1$, oriented positively, and let Γ denote the image of C in the w-plane under the map $w = f(z)$. Compute the winding number of Γ around 0 in the w-plane.

4. Evaluate $\displaystyle\int_0^\infty \frac{\sqrt{x}}{(x+1)^2} dx.$

5. Let $p(z) = z^6 - 3z^4 + 1$. Find the number of zeros (counting multiplicity) of $p(z)$ inside the unit circle $|z| = 1$. How many of them are simple zeros?