Pg 71 4a) $f(z) = 1/z^4 \quad (z \neq 0)$

$\therefore f(r,\theta) = r^{-4} e^{-i4\theta}$

$= r^{-4}(\cos 4\theta - i \sin 4\theta)$

$\therefore \quad u(r,\theta) = r^{-4}\cos 4\theta$

$v(r,\theta) = -r^{-4}\sin 4\theta$

$u_r = -4r^{-5}\cos 4\theta \qquad v_r = 4r^{-5}\sin 4\theta$

$u_\theta = -4r^{-4}\sin 4\theta \qquad v_\theta = -4r^{-4}\cos 4\theta$

$\therefore \quad r u_r = -4r^{-4}\cos 4\theta = v_\theta$

$u_\theta = -4r^{-4}\sin 4\theta = -r v_r$

Thus f is complex differentiable for $z \neq 0$

pg 76 1c) $f(x,y) = e^{-y}\sin x - i e^{-y}\cos x$

$\therefore \quad u(x,y) = e^{-y}\sin x$

$v(x,y) = -e^{-y}\cos x$

$u_x = e^{-y}\cos x \qquad v_x = +e^{-y}\sin x$

$u_y = -e^{-y}\sin x \qquad v_y = e^{-y}\cos x$

$\therefore \quad u_x = v_y \qquad u_y = -v_x$

$\therefore$ f is complex differentiable for all (x,y) and hence, entire.

pg76 1 d) $f(z) = (z^2-2)e^{-x}e^{-iy}$

$f(x,y) = (x^2-y^2-2+i2xy)(e^{-x}\cos y - ie^{-x}\sin y)$

$= [(x^2-y^2-2)e^{-x}\cos y + 2xye^{-x}\sin y]$

$+ i[2xye^{-x}\cos y - (x^2-y^2-2)e^{-x}\sin y]$

$\therefore \quad u(x,y) = (x^2-y^2-2)e^{-x}\cos y + 2xye^{-x}\sin y$

$v(x,y) = 2xye^{-x}\cos y - (x^2-y^2-2)e^{-x}\sin y$

$u_x = 2xe^{-x}\cos y - (x^2-y^2-2)e^{-x}\cos y + 2ye^{-x}\sin y - 2xye^{-x}\sin y$

$u_y = -2ye^{-x}\cos y - (x^2-y^2-2)e^{-x}\sin y + 2xe^{-x}\sin y + 2xye^{-x}\cos y$

$v_x = 2ye^{-x}\cos y - 2xye^{-x}\cos y$
$\quad - 2xe^{-x}\sin y - (x^2-y^2-2)e^{-x}\sin y$

$v_y = 2xe^{-x}\cos y - 2xye^{-x}\sin y + 2ye^{-x}\sin y - (x^2-y^2-2)e^{+x}\cos y$

$\therefore \quad u_x = v_y$

$u_y = v_x$

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2(a) $f(z) = xy + iy$

$u_x = y \qquad v_x = 0$

$u_y = x \qquad v_y = 1$

$u_x = v_y$ & $u_y = -v_x$ only at $z = i$

Since it is not complex differentiable at any other point, it is not analytic.
(It is not complex diff. in a neighbourhood of $z=i$)

pg 76 2(c) $f(z) = e^y e^{ix} = e^y \cos x + i e^y \sin x$

$u_x = -e^y \sin x \qquad v_x = e^y \cos x$

$u_y = e^y \cos x \qquad v_y = e^y \sin x$

$u_x = v_y \quad u_y = v_x$ at no points

$\therefore$ f is not analytic.

p 76 4(c) $f(z) = \dfrac{z^2+1}{(z+2)(z^2+2z+2)}$

$z+2 = 0$ for $z = -2$

$z^2+2z+2 = 0$ for $z = 1 \pm i$

$\therefore$ for $z \neq -2, 1 \pm i$

$\dfrac{1}{z^2+2z+2}, \dfrac{1}{z+2}$ are analytic

z^2+1 is entire

$\therefore$ $f(z)$ is analytic for $z \neq -2, 1 \pm i$

$\therefore$ the singular points are $-2, 1 \pm i$

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$f(z) = u + iv$

Since f is real valued $v \equiv 0$

Suppose f is analytic

then $u_x = v_y \quad u_y = -v_x$ for $z \in D$

$\Rightarrow u_x = u_y = 0$

$\Rightarrow u = \text{const}$ in D

$\therefore$ f is const.