Pratice Problems for Midterm I

Note: No books, notes, cheat sheets, calculator or any electronic devices are allowed during Midterm I exam.

1. Let $S = \{0 < |z-1| < 1\}$. Find the interior, exterior and boundary points of S. Then find the closure of S.

2. Find all cube roots of i and write them in the rectangular form $x + iy$ (Hint: $\sin\frac{\pi}{6} = \sin\frac{5\pi}{6} = \frac{1}{2}$.)

3. Let S be the horizontal line in $\mathbb{C} : S = \{\text{Im}(z) = 1\}$. Determine the image of S under the map $w = f(z) = \frac{i}{z}$.

4. Find the limits and prove by definition (using $\epsilon - \delta$ language):
(a). $\lim_{z\to 1}(z + \overline{z} - 1)$; (b). $\lim_{z\to i}(z + \overline{z} - i)$.

5. Let $f(z) = |z|^2$.

(a) Show that f is differentiable at $z = 0$.

(b). Show that f is not differentiable at any point $z \neq 0$.