

Pratice Problems for Final

Note: The final exam will be from 3:00-5:30 pm (150 minutes exam), at 0121 RWAC building on Monday, March 18th. The exam will cover everything taught from page 62-170 (sections 21-57) of the book.

Note: No books, notes, cheat sheets, calculator or any electronic devices are allowed during the final exam.

Tips: In the exam, you might be ask to write down/state (not limited to):

- Cauchy-Riemann equations in xy -coordinates, and the formula $f' = u_x + iv_x$; (You don't need to memorize the Cauchy-Riemann equations in polar form. If it will be needed in the exam, we will provide it.)
- Definition of harmonic functions, harmonic conjugates;
- Definition of exponential functions, logarithmic functions, power functions, sine and cosine functions
- Cauchy-Goursat Theorem
- Cauchy Integral formula and Cauchy Integral formula for derivatives

1. Use Cauchy-Riemann equations to prove that $f(z) = 2i \sin(\bar{z})$ is nowhere analytic.
2. Let v be a harmonic conjugate of u in \mathbb{R}^2 . Prove $-u$ is a harmonic conjugate of v in \mathbb{R}^2 .
3. (a). Compute $\int_C \cos(\frac{z}{2})dz$, where C is a contour from $z = 0$ to $\pi + 2i$.
(b). Compute $\int_C \frac{z+2}{z}dz$, where C is the contour $z = e^{i\theta} : -\frac{\pi}{4} \leq \theta \leq \frac{\pi}{4}$.
(c). Compute $\int_C \frac{z^2}{z-3}dz$ where C is the unit circle, positively oriented.
(d). Compute $\int_C \frac{1}{z^2+2z+2}dz$, where C is the unit circle, positively oriented.
- 4.(a). Compute $\int_C \frac{z^2}{z-3}dz$, where $C = \{|z| = 4\}$, positively oriented.
(b). Compute $\int_C \frac{z^2}{(z-3)^2}dz$, where $C = \{|z| = 4\}$, positively oriented.
(c). Compute $\int_C \frac{z^4}{(z-3)^5}dz$, where $C = \{|z| = 4\}$, positively oriented.
(d). Compute $\int_C \frac{z}{(z-1)(z-3)}dz$, where $C = \{|z - 1| = 1\}$, positively oriented.
(e). Compute $\int_C \frac{1}{4z^2+1}dz$, where $C = \{|z - \frac{1}{2}| = \frac{2}{3}\}$, positively oriented.
5. Problems in the Practice Exam for Midterm II.
6. Problems in Midterm II exam.
7. All related Homework problems.