1. State Liouville’s theorem.

2. Let \( f \) be an entire function. Assume the harmonic function \( u(x, y) = \text{Re} f(z) \) satisfies that \( u(x, y) \leq 2 \) for all \((x, y)\) on the plane. Prove that \( f \) is constant.

3. (a) Find the Laurent series for the function \( f(z) = \frac{1}{z^2(1-z)} \) on \( \{ 0 < |z| < 1 \} \).

   (b) Show that when \( 0 < |z-1| < 2 \),
   
   \[
   \frac{z}{(z-1)(z-3)} = -3 \sum_{n=0}^{\infty} \frac{(z-1)^n}{2^{n+2}} - \frac{1}{2(z-1)}.
   \]

4. Let \( f(z) = \frac{e^z}{z^2+1} \). Find \( f^{(3)}(0) \) and \( f^{(4)}(0) \).

5. (a) Compute \( \int_C \cos(z/2) \, dz \), where \( C \) is a contour from \( z = 0 \) to \( \pi + 2i \).

   (b) Compute \( \int_C \frac{z+2}{z} \, dz \), where \( C \) is the contour \( z = e^{i\theta} : -\frac{\pi}{4} \leq \theta \leq \frac{\pi}{4} \).

   (c) Compute \( \int_C \frac{z^2}{z-3} \, dz \) where \( C \) is the unit circle, positively oriented.

   (d) Compute \( \int_C \frac{1}{z^2+2z+2} \, dz \), where \( C \) is the unit circle, positively oriented.

6. (a) Compute \( \int_C \frac{z^2}{z-3} \, dz \), where \( C = \{ |z| < 4 \} \), positively oriented.

   (b) Compute \( \int_C \frac{z^2}{(z-1)^2} \, dz \), where \( C = \{ |z| < 4 \} \), positively oriented.

   (c) Compute \( \int_C \frac{z}{(z-1)(z-3)} \, dz \), where \( C = \{ |z-1| = 1 \} \), positively oriented.

   (d) Compute \( \int_C \frac{1}{4z^2+1} \, dz \), where \( C = \{ |z - \frac{1}{2}| = \frac{2}{3} \} \), positively oriented.

7. Let \( f(z) = \frac{1}{z(z-2)} \).

   (a) Find the residue of \( f \) at 0.

   (b) Find the residue of \( f \) at 1.

   (c) Find the residue of \( f \) at 2.