Online Exam Instruction

Here is the instruction for the online final exam (4:00-6:20pm, PT) on March 19th.

1. It will be an open-book exam. You can only use the book, lecture videos, lecture notes and HW solutions (of your own and those we posted on course website/canvas). Any other resources/assistant are not allowed. In particular, any other resources online cannot be used and you may not collaborate or communicate (in-person or online or in any other form) with any other humans while working on this exam. Any sign of cheating/collaboration observed in the grading will be reported immediately to the Academic Integrity Office.

2. On the next page, you will see an integrity pledge. You will need to sign and date for the pledge. You don’t need to print the pledge page out. You can just copy the line “I excel with integrity” on your own paper and then sign and date it. I attach the pledge page so you can take a look in advance.

3. If you have any questions about any problem in the exam, we encourage you to use your own judgment and understanding (Of course we do our best to formulate each problem in a clean way to avoid ambiguity). If you still feel that you need to ask, then please email me (m3xiao@ucsd.edu) and cc Soumya (s1gangul@eng.ucsd.edu).

4. You don’t need to print the exam out. You can just write your solutions on blank papers or latex your solutions or write on a tablet. You will need to finish and upload your solutions (handwriting scanned file, or latex generated pdf, tablet writing pdf files, or any format that is accepted by Gradescope) to Gradescope by 6:20pm, PT, on March 19th.

5. When you upload your exam solution, Gradescope will ask you to match each of the problems with the page on which your solution for that problem appears. Please do it carefully and make sure they are well-matched.

6. Your submission by Gradescope is sufficient. But if for any reason you cannot upload the exam to Gradescope, then please email your exam to me (m3xiao@ucsd.edu) and cc Soumya (s1gangul@eng.ucsd.edu) by 6:25pm (pdf format strongly preferred when you send by email). We will decide what to do with your exam (Do not send email if you successfully upload the exam to Gradescope). If you fail to upload via Gradescope and also fail to email your exam by 6:25pm, PT, on March 19th, then your exam will not be accepted.

7. Important Tips: You are strongly recommended to leave enough time for yourself to scan and upload the exam, don’t wait until the last minute.

8. We will look for any possible signs of academic misconduct during grading the exam. If any suspicious signs are spotted, then the students may be asked to a follow-up Zoom meeting in which they will be asked to justify their work on the exam and show that it was their own work. If the follow-up is unconvincing, or the student is unable or unwilling to engage, their exam will be forwarded to the AI Office for a potential violation.
Excel with Integrity Pledge

The Excel with Integrity pledge affirms the UC San Diego commitment to excel with integrity both on and off campus, in academic, professional, and research endeavors.

According to the International Center for Academic Integrity, academic integrity means having the courage to act in ways that are honest, fair, responsible, respectful & trustworthy even when it is difficult. Creating work with integrity is important because otherwise we are misrepresenting our knowledge and abilities and the University is falsely certifying our accomplishments. And when this happens, the UCSD degree loses its value and we’ve all wasted our time and talents!

Student Name: ____________________________________________

Excel with Integrity Pledge

I understand I may not collaborate or communicate with any other humans while working on this exam.

I am fair to my classmates and instructors by not using any unauthorized aids.

I respect myself and my university by upholding educational and evaluative goals.

I am honest in my representation of myself and of my work.

I accept responsibility for ensuring my actions are in accord with academic integrity.

I show that I am trustworthy even when no one is watching.

Affirm your adherence to this pledge by writing the following statement in the space below:

I Excel with Integrity.

Signature: ____________________________________________ Date: ____________
Math 120A Final Exam

The exam starts here: The exam has 103 points in total, but we will grade it out of 100 points. That means we will add up all the points that you get, but the maximum total points you can have is 100.

(1) 0. Make sure you have read the exam instruction and the integrity pledge. Then copy the line “I excel with integrity” and then sign and date. You don’t need to do this on a separated paper. It can be the same papers where you are going to do the remaining questions.

(8) 1 (a). First state the definition of sin \( z \) for a complex variable \( z \). Then use it to compute \( \sin(i) \). Simply your answer. You can express your answer in terms of \( e \) and \( \frac{1}{e} \). Show your work.

(6) 1 (b). Compute the principal value of \((-i)^i\). Simply your answer. Your final answer can involve \( e \) and \( \pi \). Show your work.
2. Let $f$ be an entire function. Write $v(x, y)$ for the imaginary part of $f(z)$, i.e., $v(x, y) = \text{Im} f(z)$. Assume $v(x, y) \leq 2$ for all $(x, y)$ on the plane. Prove that $f$ is a constant function.
(6)  3. (a) Compute \( \int_C \frac{1}{z} \, dz \), where \( C \) is a contour from \( z = -i \) to \( z = 1 \) and does not pass through 0. Show your work.

(7)  3. (b) Compute \( \int_C \frac{e^z}{z^9} \, dz \), where \( C \) is the circle \( |z - 2| = 3 \) (positively oriented). Show your work.
4. (a) Compute \( \int_{C} \frac{z^{10}}{z^{2} - 5z + 6} \, dz \), where \( C \) is the circle \( |z - 1| = \frac{1}{2} \). Show your work.

4. (b) Compute \( \int_{C} \frac{z}{z^{2} + 4} \, dz \), where \( C \) is the circle \( |z - i| = 2 \) (positively oriented). Show your work.
(9) 5 (a). Find the Laurent series of \( f(z) = \frac{1}{z+1} + \frac{1}{2-z} \) on the domain \( \{ 1 < |z| < 2 \} \). Show your work.

(9) 5 (b). Let \( f(z) = \frac{z^2}{z+1} \). Use Taylor series to compute \( f^{(3)}(0) \) and \( f^{(4)}(0) \). Show your work.
\( \text{(10) 6 (a). Let } f(z) = \frac{1}{z^2(z-2)^2}. \text{ First find the residue of } f(z) \text{ at } z = 0. \text{ Then find the residue of } f(z) \text{ at } z = 2. \text{ Show your work.} \)

\( \text{(5) 6 (b). Let } f(z) = \frac{1}{z^2(z-2)^2}. \text{ Compute } \int_C f(z)dz, \text{ where } C \text{ is the circle } |z-4| = 5 \text{ (positively oriented). You can use your answer in part (a). Make sure you mention the name of the theorem that you use.} \)
(9) 7. Multiple choice problem. Write your answer in the parenthesis. There is only one correct answer for every question. **You don’t need to justify.** If you write solutions in your own papers, you should write in the format like “(1) A; (2) B; (3) C.”

(1). Which of the following statement is true? ( )

(A). The function \( f(z) = \tan z \) is not entire.

(B). \( f(z) = \frac{1}{z-1} \) has no antiderivative on the disk \( D = \{|z| < 1\}; \)

(C). Let \( f(z) = \frac{1}{z+1} \). Then residue of \( f \) at 0 equals 1;

(D). It holds that \( \Log(z^2) = 2\Log(z) \) for every nonzero complex number \( z \).

(2). What is the value of \((-1)^\frac{1}{i}\)? ( )

(A). \( e^{2ni+1}, n \in \mathbb{Z} \); (B). \( e^{n+i}, n \in \mathbb{Z} \); (C). \( e^{(2n+1)i}, n \in \mathbb{Z} \); (D). \( e^{2n+i}, n \in \mathbb{Z} \).

(3). Which of the following statement is false? ( )

(A). The function \( f(z) = \Log z \) is not analytic on \( \{z \in \mathbb{C} : z \neq 0\} \).

(B). \( |\sin z| \leq 1 \) for all complex number \( z \);

(C). Let \( f \) be an entire function such that \( |f(z)| \geq 1 \) everywhere on \( \mathbb{C} \). Then \( f \) is a constant function;

(D). \( \sin(z + 2\pi) = \sin z \) for all complex number \( z \).
(6) 8. Compute \( \int_{C} \frac{1}{z} \cos \left( \frac{1}{z} \right) dz \), where \( C \) is the circle \(|z - 2| = 3\) (positively oriented). Show your work.
Math 120A Final Exam

Bonus Question. The question 9 is worth only 3 points.

(3) 9. Write down the theorem you learned in Math 120A that impressed you most. State the theorem. Then explain why the theorem is impressive to you. You will get points if what you write makes sense to us.

The exam ends here.